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# Unbalance as excitation force

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Dynamics and Oscillations

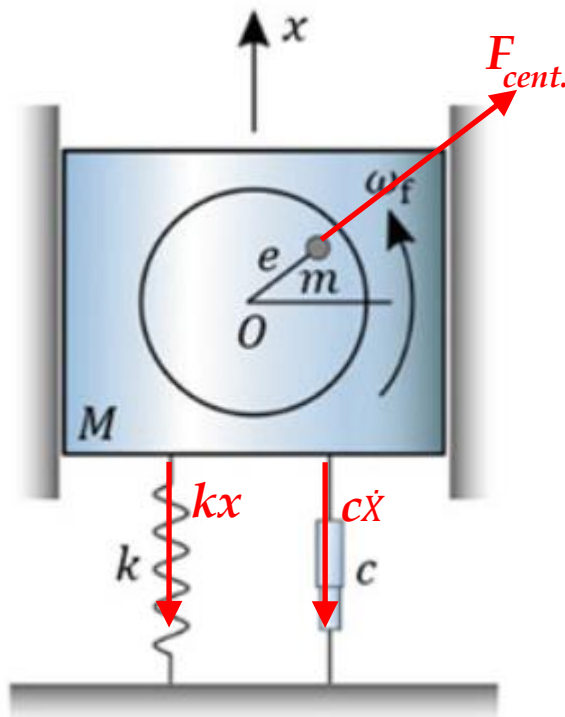
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# Centrifugal force



Material mass  $m$  that rotates about point  $O$  with angular velocity  $\omega$  with excentre  $e$  generates centrifugal force that could be defined by equation:

$$F_{cent.} = m \cdot \Omega^2 \cdot e$$

This force direction is changing along  $x$  direction and which depends on current position of mass  $m$ . Current position could be described by angle  $\theta$  that could be written (in case where angular velocity is constant)  $\theta = \theta_0 + \omega t$ . Equation of motion could be written as:

$$F_{cent.} \sin(\Omega t + \theta_0) - kx - c\dot{x} = M\ddot{x}$$

$$M\ddot{x} + kx + c\dot{x} = F_{cent.} \sin(\Omega t + \theta_0)$$

$$M\ddot{x} + kx + c\dot{x} = m \cdot \omega^2 \cdot e \cdot \sin(\Omega t + \theta_0)$$

Solution of equation of motion:

$$x_p(t) = \frac{X_0}{\sqrt{(1-r^2)^2 + (2r\xi)^2}} \sin(\Omega t + \theta_0) \quad \text{or} \quad x_p(t) = \left(\frac{m}{M} e\right) \beta_r \sin(\Omega t + \theta_0)$$

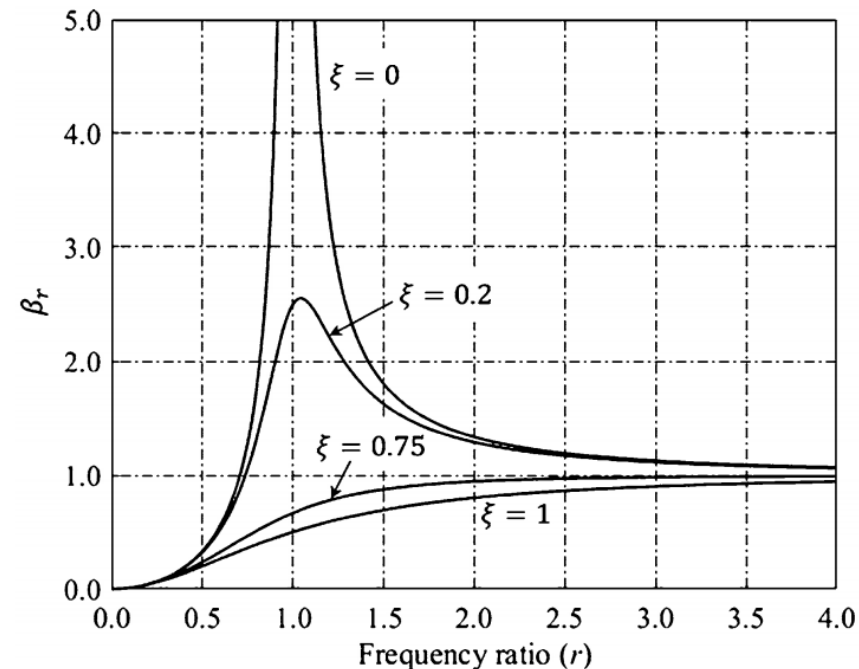
$$\beta_r = \frac{\lambda^2}{\sqrt{(1-\lambda)^2 + (2r\xi)^2}}$$

where

$$\xi = \frac{c}{2m\Omega}$$

$$\lambda = \frac{\Omega}{\omega}$$

$$X_0 = \frac{m \cdot \omega^2 \cdot e}{k} = \left(\frac{m}{M} e\right) \lambda^2$$

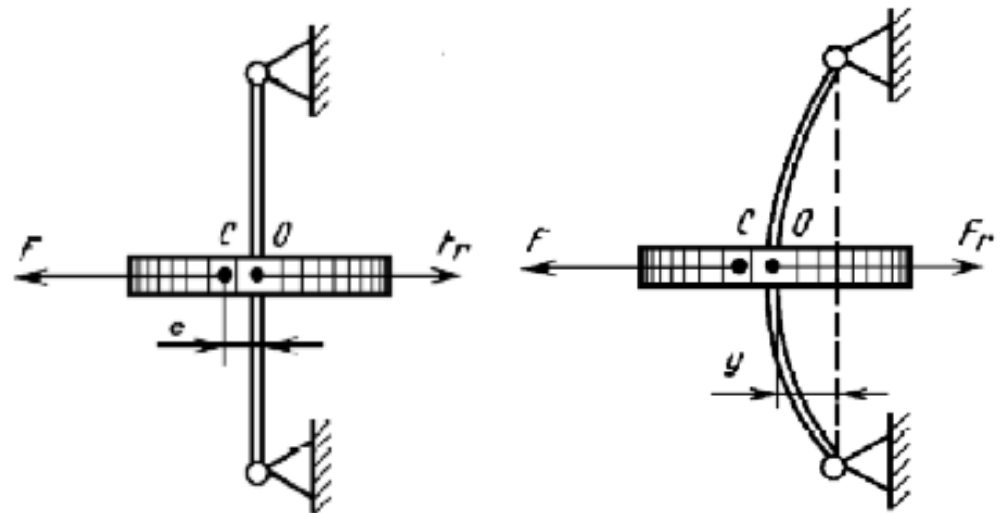


## Unbalance of rotating parts of machines

Some of consequences of unbalance are additional dynamic loads, vibrations and noise that are transferred to the bearings, casing, pedestal or foundations of machinery.

### Unbalance causes:

- constructional - functional
- material non-homogeneity
- Inaccurate production
- assembly errors
- during exploitation
- after the maintenance (repairs)



## Balance conditions

If rotor is balanced, it is required that main vector and main moment of inertial forces is equal to zero.

$$F_x = Mx_s \omega^2 = 0$$

$$M_{ox} = -I_{yz} \omega^2 = 0$$

$$F_y = My_s \omega^2 = 0$$

$$M_{oy} = -I_{xz} \omega^2 = 0$$

$$F_z = 0$$

$$M_{oz} = 0$$

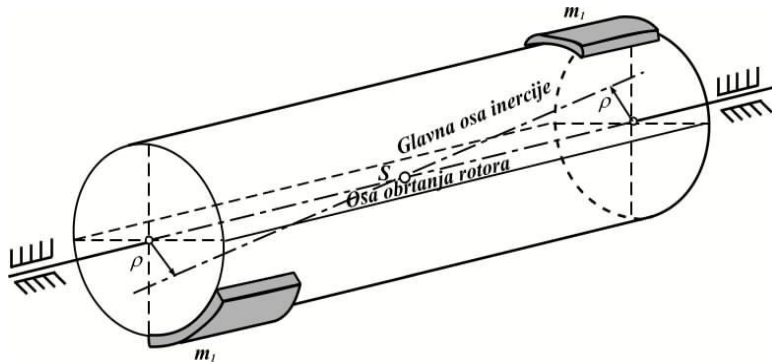
$$\left. \begin{array}{l} x_s = 0 \\ y_s = 0 \end{array} \right\}$$

*Mass center has to be on its rotational axis.*

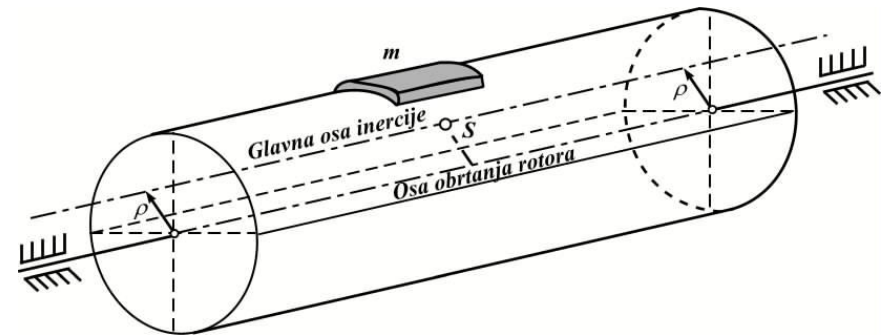
$$\left. \begin{array}{l} I_{xz} = 0 \\ I_{yz} = 0 \end{array} \right\}$$

*Rotational axis has to be aligned with main inertia axis.*

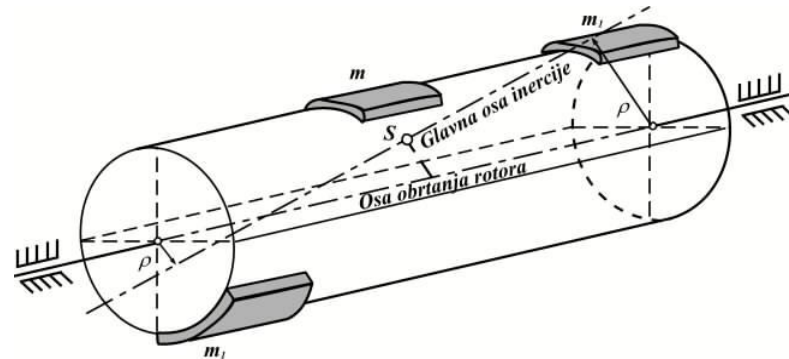
# Unbalance types



**Dynamical unbalance**



**Statical unbalance**

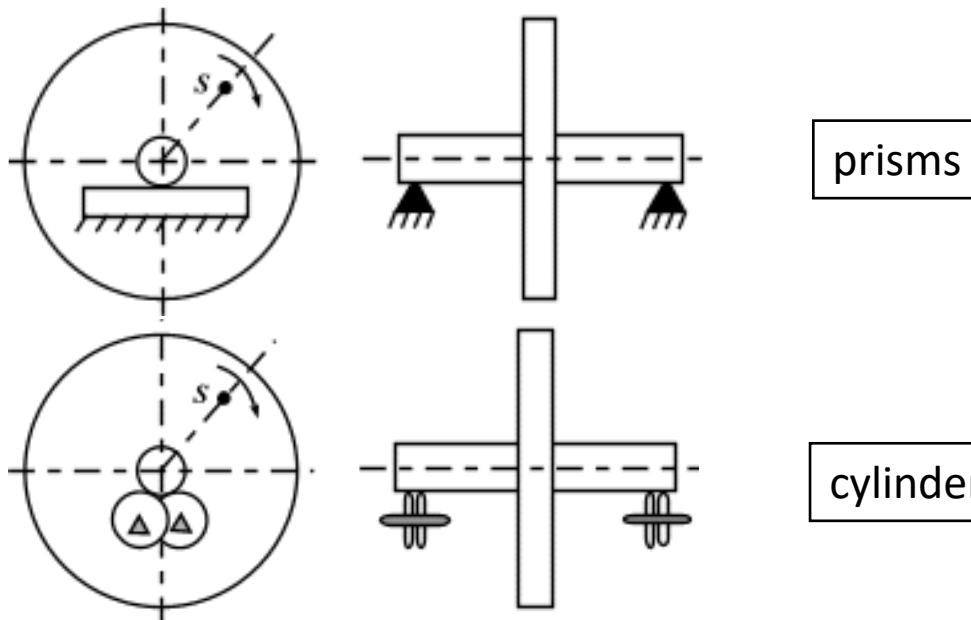
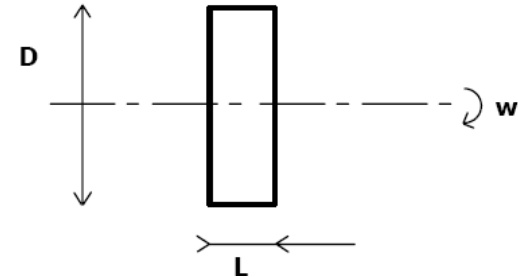


**Combined unbalance**

Statical balancing is used only for rotors shaped as disks (large diameter, small thickness).

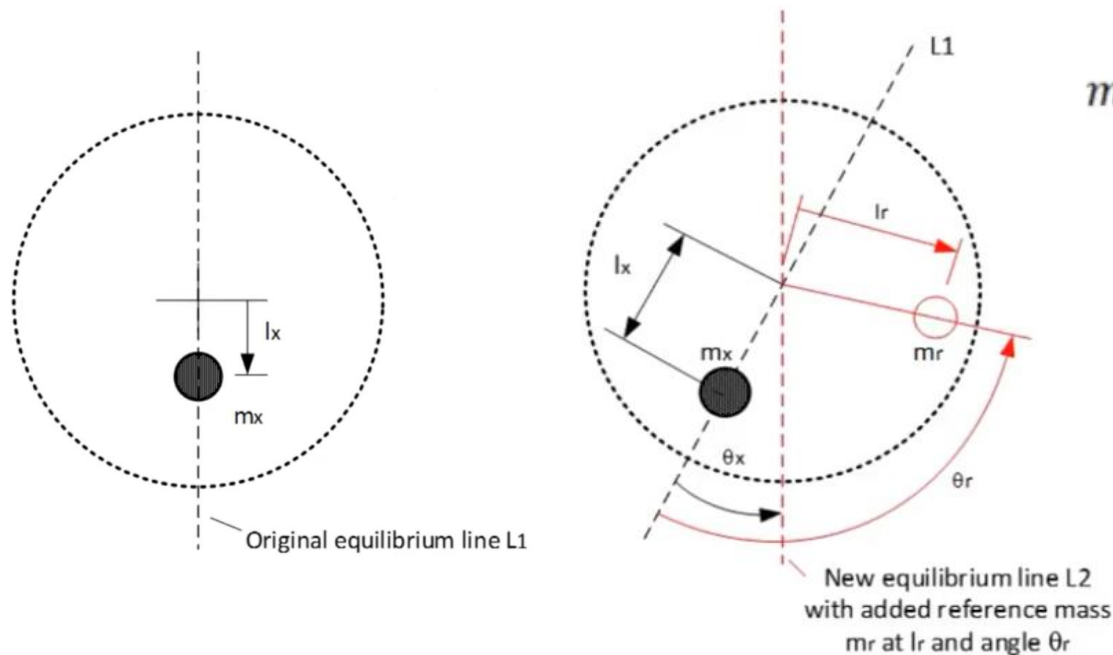
It is required to evaluate distance between the supports additionally to the dimensions of rotor.

$L/D < 0.5$   
and  
 $w < 1000 \text{ tr/mn}$



In the dependance of sizes and positions of shaft sleeves, it could be chosen that balancing is done on prisms or cylinders.

## Statical balancing – clear unbalance



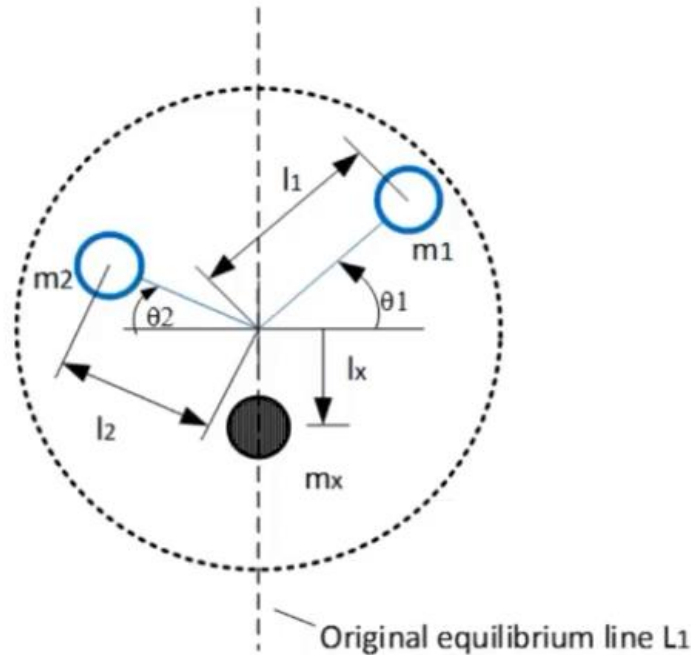
$$m_x g l_x \sin \theta_x - m_r g l_r \sin(\theta_r - \theta_x) = 0$$

$$m_x l_x = \frac{m_r l_r \sin(\theta_r - \theta_x)}{\sin \theta_x}$$

**Heavy spot will take  
this position - L1.**

**Mass  $m_r$  is added on the  
distance  $l_r$  rotated by  $\theta_r$  from  
L1. New direction (vertical)  
L2 is marked and angle  $\theta_x$  is  
measured.**

## Statical balancing – clear unbalance



*Balancing mass could be divided in two or more masses, if required by construction of rotor.*

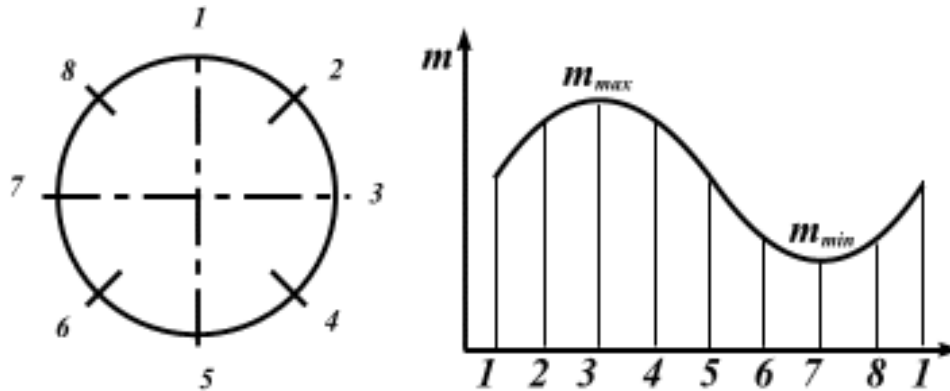
$$m_1 l_1 \cos \theta_1 - m_2 l_2 \cos \theta_2 = 0$$

$$m_1 l_1 \sin \theta_1 + m_2 l_2 \sin \theta_2 = m_x l_x$$

$$m_1 = \frac{m_x l_x \cos \theta_2}{l_1 \sin(\theta_1 + \theta_2)}$$

$$m_2 = \frac{m_x l_x \cos \theta_1}{l_2 \sin(\theta_1 + \theta_2)}$$

## Staical balancing – unclear unbalance



- Front side of rotor is divided in 6 to 8 equal parts;
- In every point you should add the same mass so rotor is rotated by the same angle (usually 10-15deg).
- From the created diagram it could be concluded that heavy spot is in the direction of minimal and maximal mass. Trial weight is added to the max point.

$$m_p = \frac{(m_{\max} + m_{\min})}{2}$$

Counterweight mass

## Car wheel balancing

- Balancing of tyres is process that is used to reduce wear, increase comfort in the vehicle and reduce loads on the car parts.
- Unbalance could appear because of the tire damage, deformations of tires or wheel, unequal wear of tire or similar.



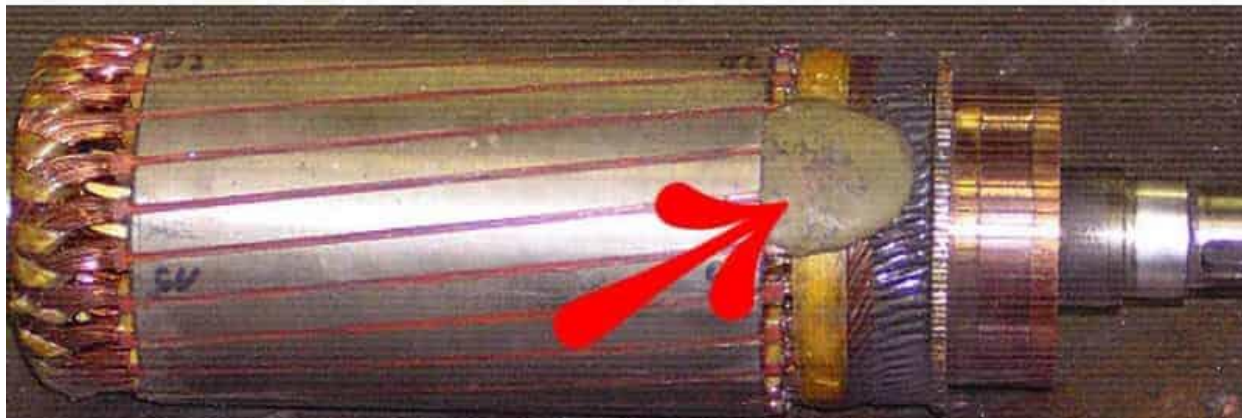
## Balancing machine – for vehicle tires



## Electromotor rotor balancing



Removal of Material by Cutting the The Core for Rotor Balance



## Machine for statical balancing of rotors of electromotor

