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Unbalance as excitation force

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Dynamics and Oscillations

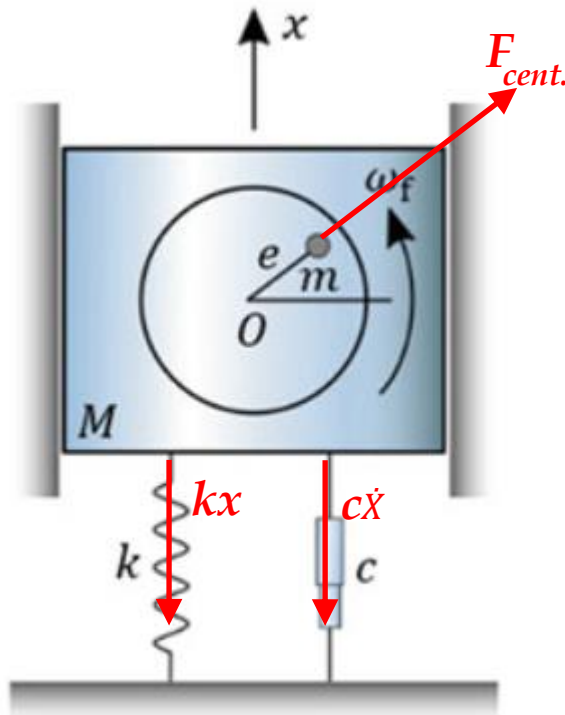
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Centrifugal force



Material mass m that rotates about point O with angular velocity ω with excentre e generates centrifugal force that could be defined by equation:

$$F_{cent.} = m \cdot \Omega^2 \cdot e$$

This force direction is changing along x direction and which depends on current position of mass m . Current position could be described by angle θ that could be written (in case where angular velocity is constant) $\theta = \theta_0 + \omega t$. Equation of motion could be written as:

$$F_{cent.} \sin(\Omega t + \theta_0) - kx - c\dot{x} = M\ddot{x}$$

$$M\ddot{x} + kx + c\dot{x} = F_{cent.} \sin(\Omega t + \theta_0)$$

$$M\ddot{x} + kx + c\dot{x} = m \cdot \omega^2 \cdot e \cdot \sin(\Omega t + \theta_0)$$

Solution of equation of motion:

$$x_p(t) = \frac{X_0}{\sqrt{(1-r^2)^2 + (2r\xi)^2}} \sin(\Omega t + \theta_0) \quad \text{or} \quad x_p(t) = \left(\frac{m}{M}e\right) \beta_r \sin(\Omega t + \theta_0)$$

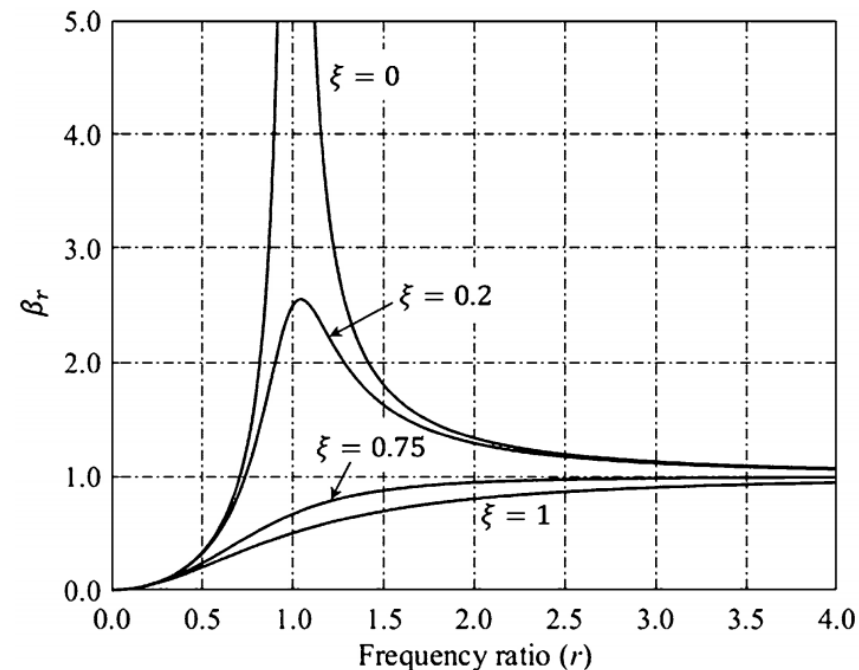
$$\beta_r = \frac{\lambda^2}{\sqrt{(1-\lambda)^2 + (2r\xi)^2}}$$

where

$$\xi = \frac{c}{2m\Omega}$$

$$\lambda = \frac{\Omega}{\omega}$$

$$X_0 = \frac{m \cdot \omega^2 \cdot e}{k} = \left(\frac{m}{M}e\right) \lambda^2$$

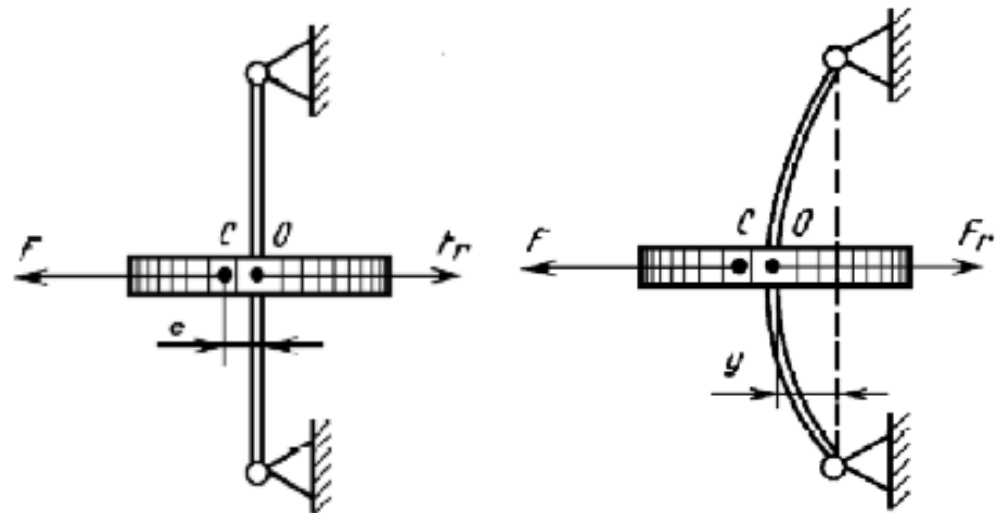


Unbalance of rotating parts of machines

Some of consequences of unbalance are additional dynamic loads, vibrations and noise that are transferred to the bearings, casing, pedestal or foundations of machinery.

Unbalance causes:

- constructional - functional
- material non-homogeneity
- Inaccurate production
- assembly errors
- during exploitation
- after the maintenance (repairs)



Balance conditions

If rotor is balanced, it is required that main vector and main moment of inertial forces is equal to zero.

$$F_x = Mx_s \omega^2 = 0$$

$$M_{ox} = -I_{yz} \omega^2 = 0$$

$$F_y = My_s \omega^2 = 0$$

$$M_{oy} = -I_{xz} \omega^2 = 0$$

$$F_z = 0$$

$$M_{oz} = 0$$

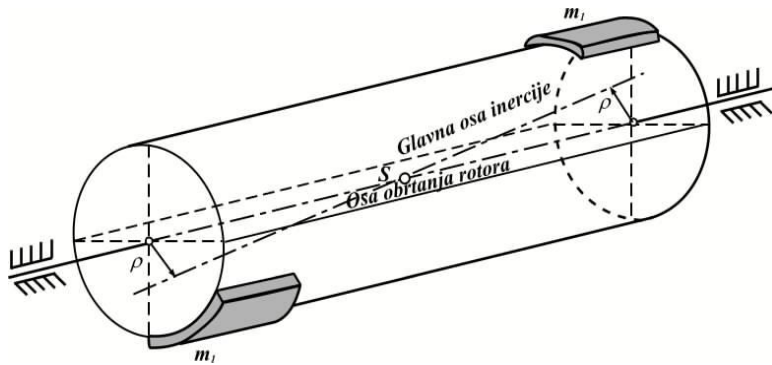
$$\left. \begin{array}{l} x_s = 0 \\ y_s = 0 \end{array} \right\}$$

Mass center has to be on its rotational axis.

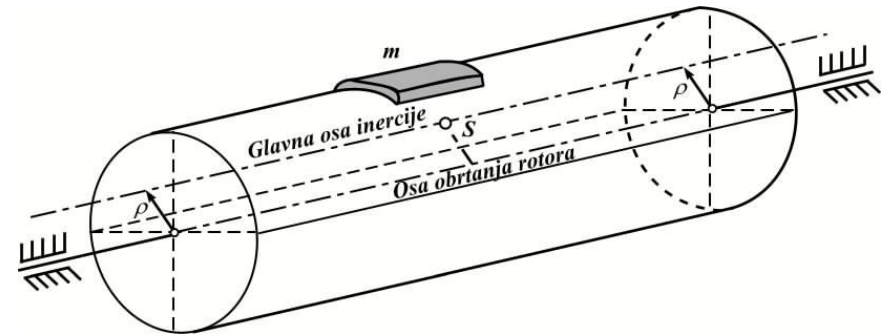
$$\left. \begin{array}{l} I_{xz} = 0 \\ I_{yz} = 0 \end{array} \right\}$$

Rotational axis has to be aligned with main inertia axis.

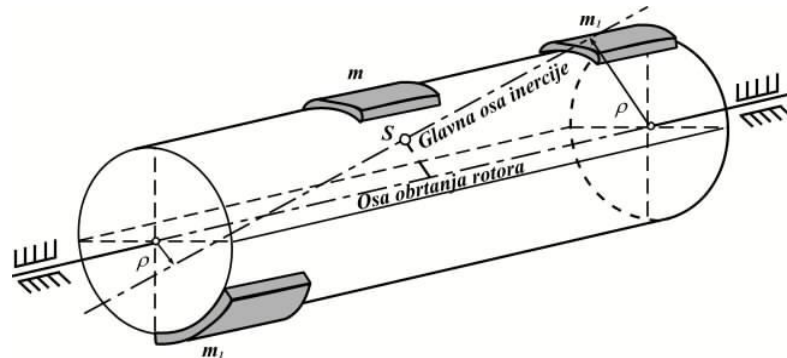
Unbalance types



Dynamical unbalance



Static unbalance

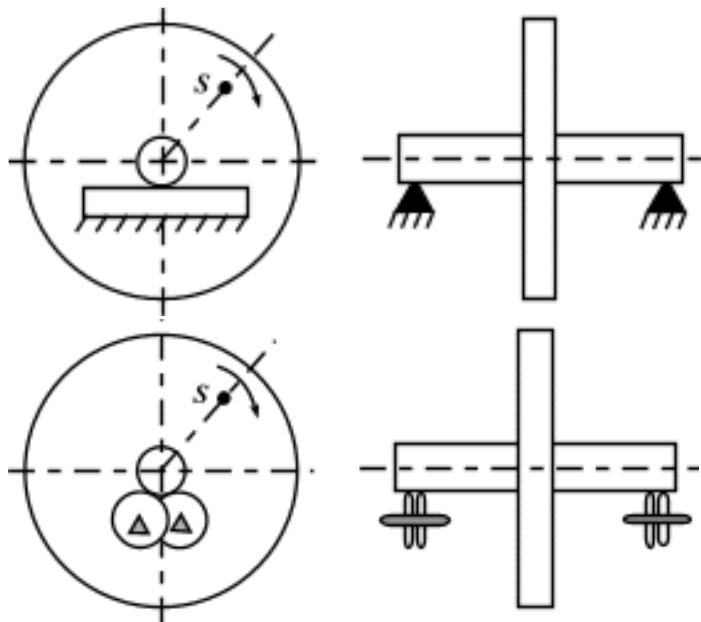
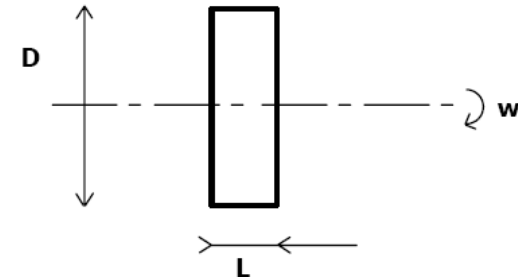


Combined unbalance

Statical balancing is used only for rotors shaped as disks (large diameter, small thickness).

It is required to evaluate distance between the supports additionally to the dimensions of rotor.

$L/D < 0.5$
and
 $w < 1000 \text{ tr/mn}$

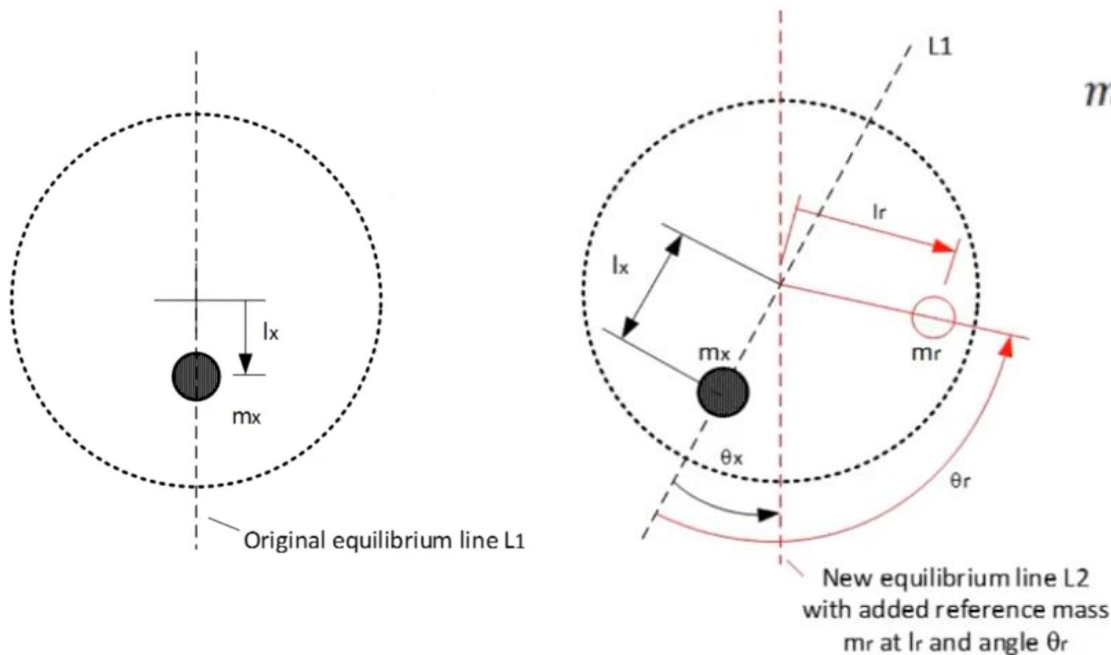


prisms

cylinders

In the dependance of sizes and positions of shaft sleeves, it could be chosen that balancing is done on prisms or cylinders.

Statical balancing – clear unbalance



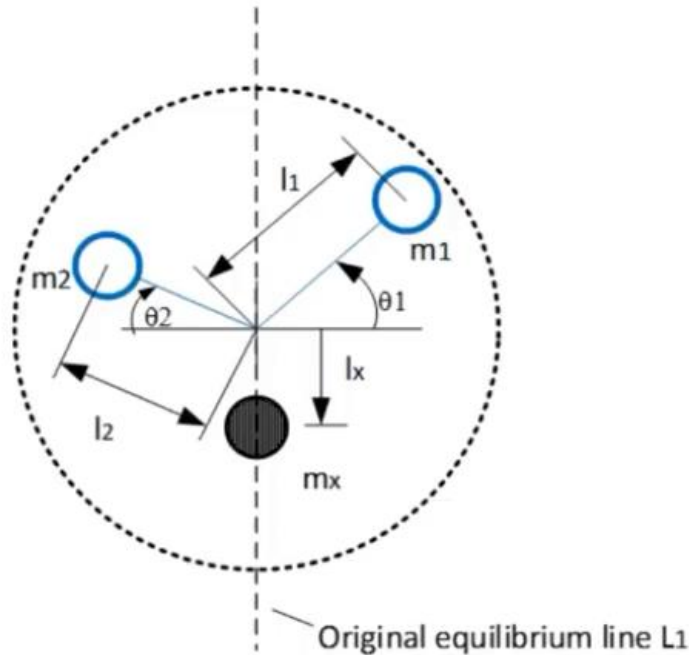
$$m_x g l_x \sin \theta_x - m_r g l_r \sin(\theta_r - \theta_x) = 0$$

$$m_x l_x = \frac{m_r l_r \sin(\theta_r - \theta_x)}{\sin \theta_x}$$

*Heavy spot will take
this position - L1.*

Mass m_r is added on the
distance l_r rotated by θ_r from
L1. New direction (vertical)
L2 is marked and angle θ_x is
measured.

Statical balancing – clear unbalance



Balancing mass could be divided in two or more masses, if required by construction of rotor.

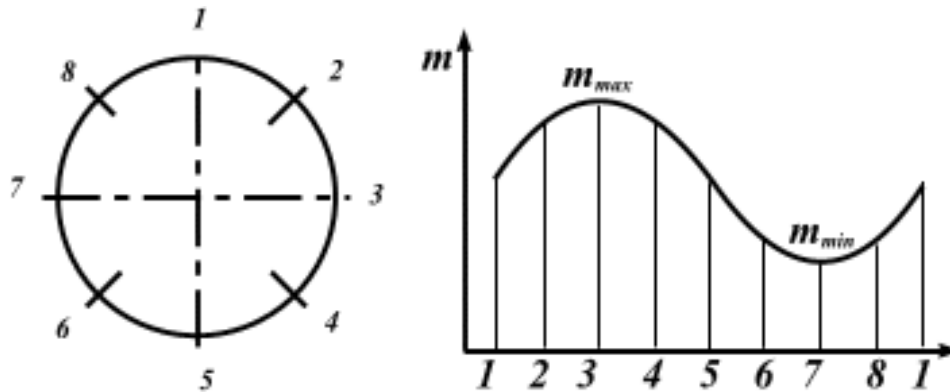
$$m_1 l_1 \cos \theta_1 - m_2 l_2 \cos \theta_2 = 0$$

$$m_1 l_1 \sin \theta_1 + m_2 l_2 \sin \theta_2 = m_x l_x$$

$$m_1 = \frac{m_x l_x \cos \theta_2}{l_1 \sin(\theta_1 + \theta_2)}$$

$$m_2 = \frac{m_x l_x \cos \theta_1}{l_2 \sin(\theta_1 + \theta_2)}$$

Staical balancing – unclear unbalance



- Front side of rotor is divided in 6 to 8 equal parts;
- In every point you should add the same mass so rotor is rotated by the same angle (usually 10-15deg).
- From the created diagram it could be concluded that heavy spot is in the direction of minimal and maximal mass. Trial weight is added to the max point.

$$m_p = \frac{(m_{\max} + m_{\min})}{2}$$

Counterweight mass

Car wheel balancing

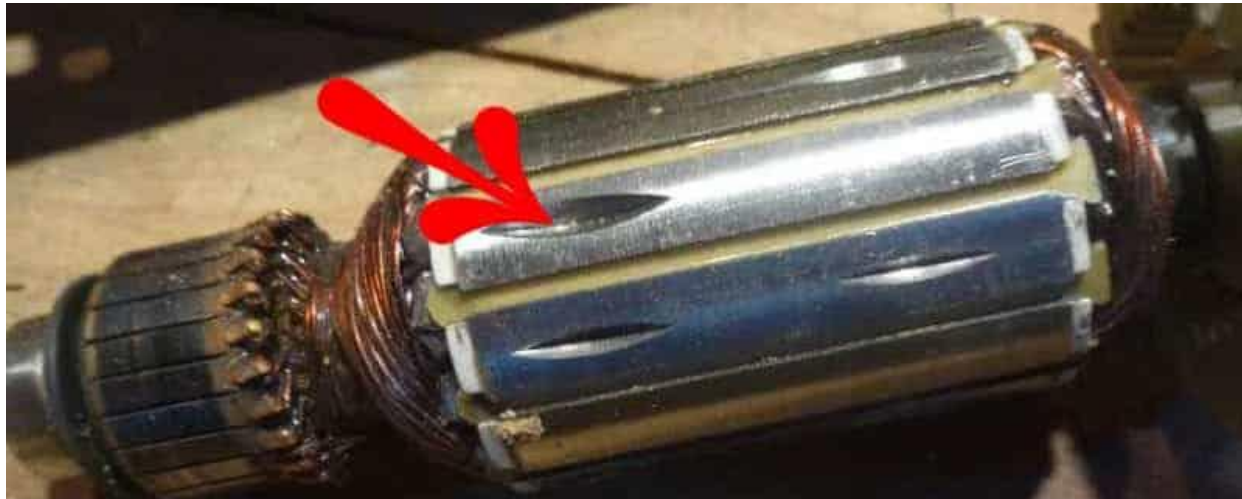
- Balancing of tyres is process that is used to reduce wear, increase comfort in the vehicle and reduce loads on the car parts.
- Unbalance could appear because of the tire damage, deformations of tires or wheel, unequal wear of tire or similar.



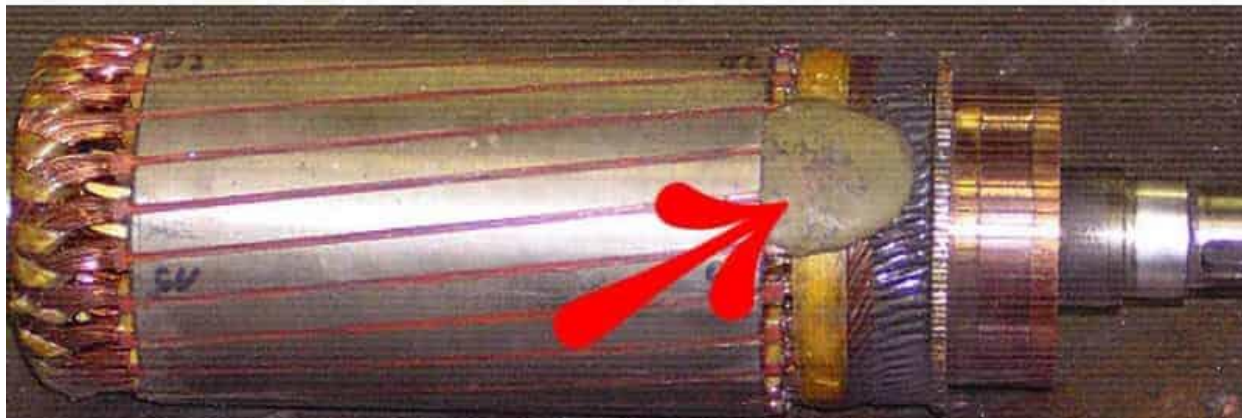
Balancing machine – for vehicle tires



Electromotor rotor balancing



Removal of Material by Cutting the The Core for Rotor Balance



Machine for statical balancing of rotors of electromotor

