HARMONICALLY EXCITED VIBRATIONS

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Dynamics and Oscilations

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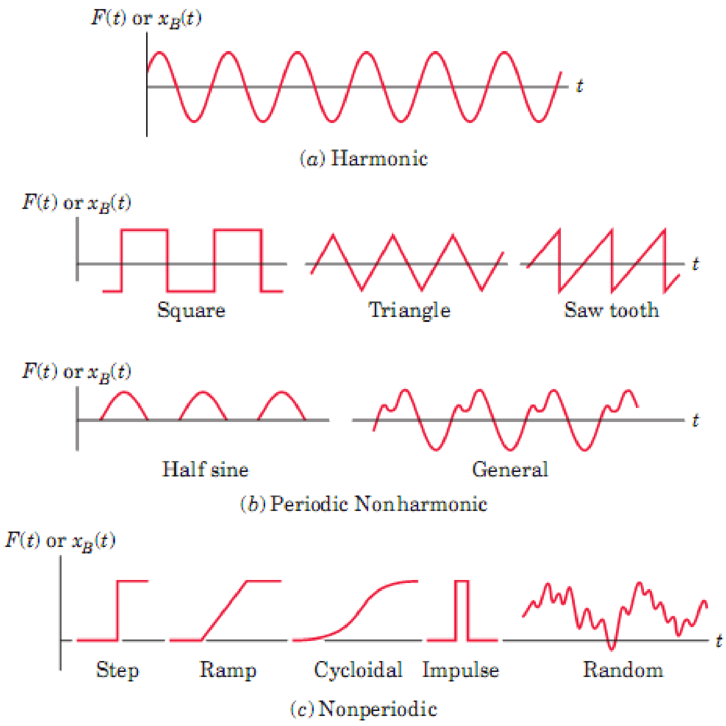
Types of disturbance forces



If there is present some disturbance force that impacts on free harmonic vibration motion, then that motion is called FORCED VIBRATION.

DISTURBANCE FORCE TYPES:

- ***Harmonic***
- ***Periodical nonharmonic***
- ***Nonperiodic***

Equation of motion of harmonically excited vibrations

$$m\ddot{x} = -kx + F_0 \sin(\Omega t + \beta)$$

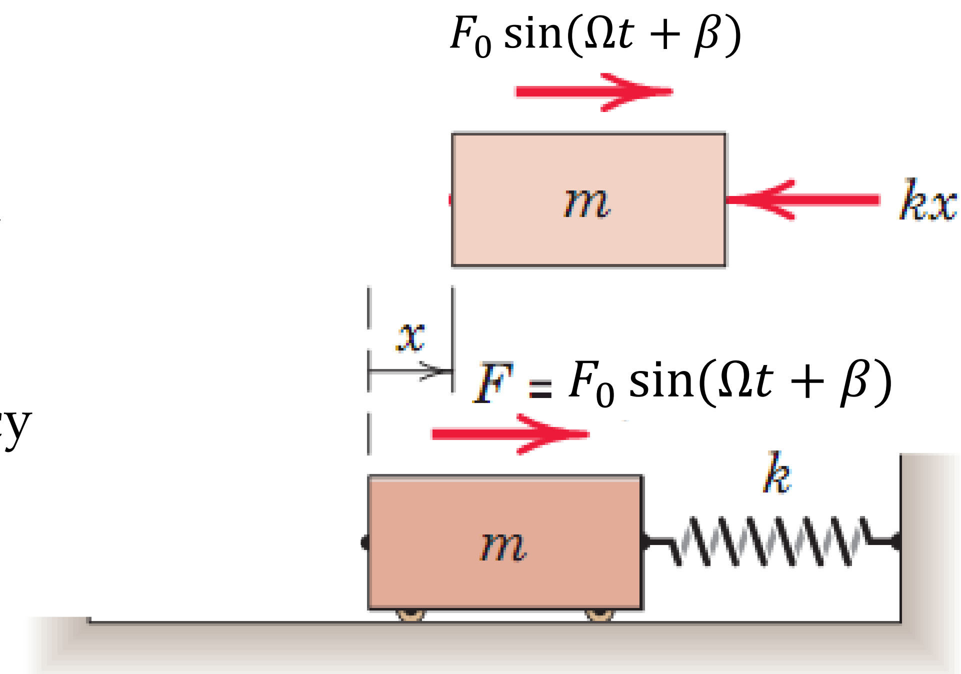
$$m\ddot{x} + kx = F_0 \sin(\Omega t + \beta)$$

$$\ddot{x} + \omega^2 x = h \sin(\Omega t + \beta)$$

Differential equation of harmonically excited vibrations

Ω – exciting force natural frequency
F_0 – exciting force amplitude
β – exciting force phase shift

$$h = \frac{F_0}{m}$$

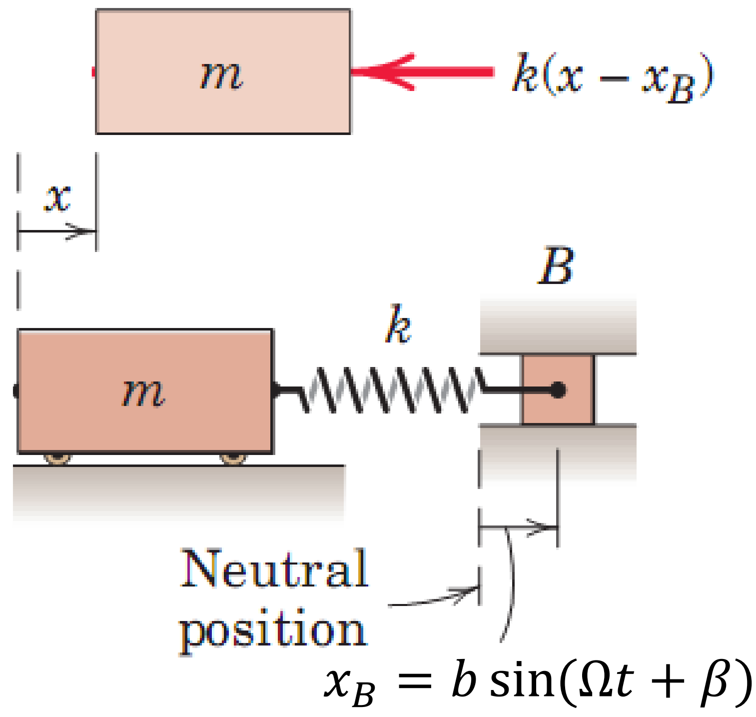


Harmonic motion of base

$$m\ddot{x} = -k(x - x_B)$$

$$\ddot{x} + \omega^2 x = \frac{kb}{m}\sin(\Omega t + \beta)$$

Differential equation of undamped forced vibration – base motion case



b – base movement amplitude

Solution of the equation of motion

$$x = x_h + x_p$$

$$x_h = C_1 \cos \omega t + C_2 \sin \omega t$$

$$\omega \neq \Omega$$

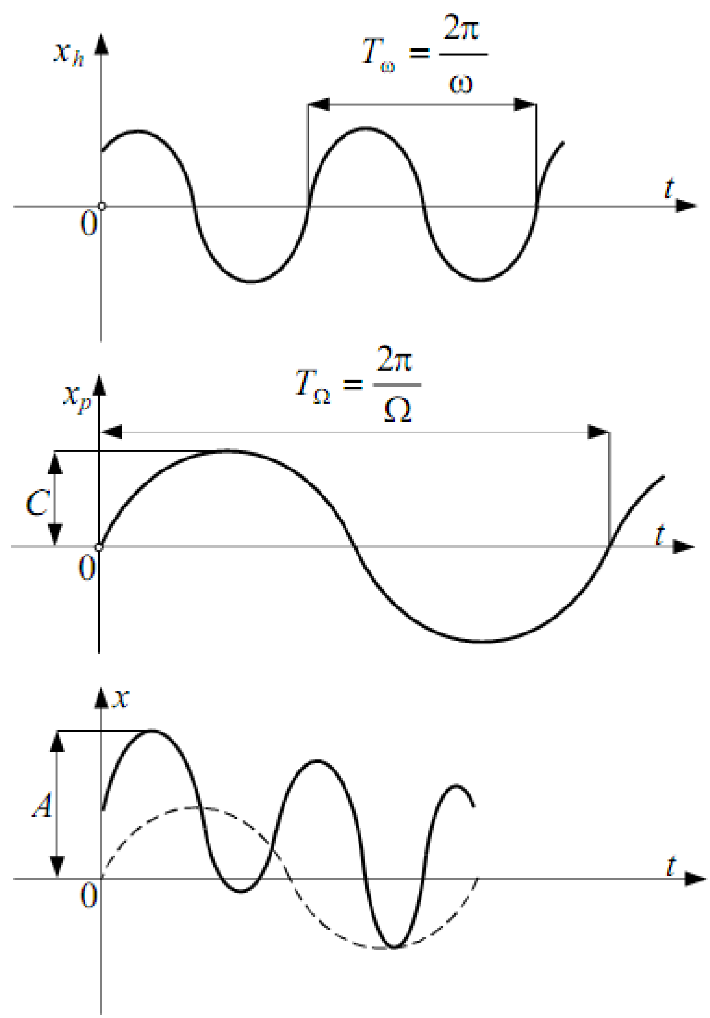
$$x_p = C \sin(\Omega t + \beta) \quad \textit{where} \quad C = \frac{h}{\omega^2 - \Omega^2}$$

$$x_p = \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$

$$x = C_1 \cos \omega t + C_2 \sin \omega t + \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$

or

$$x = A \sin(\omega t + \alpha) + \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$

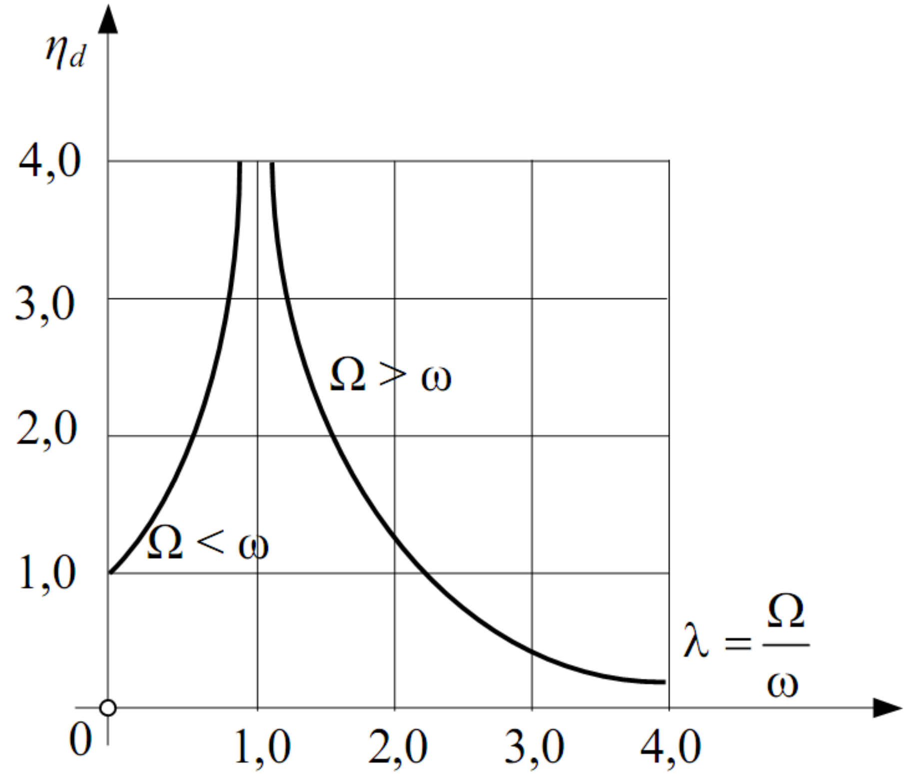


Resonance (Ω = ω)

$$x_p = \frac{\frac{h}{\omega^2}}{1-\left(\frac{\Omega}{\omega}\right)^2}\sin(\Omega t + \beta)$$

Statical spring elongation $\frac{h}{\omega^2} = \frac{m \cdot h}{c} = f_{st}$

Disturbance coefficient $\lambda = \frac{\Omega}{\omega}.$



Amplification factor

$$\eta_d = \frac{C_d}{C_{st}} = \frac{\frac{h}{\omega^2 - \Omega^2}}{f_{st}} = \frac{\omega^2}{\omega^2 - \Omega^2} = \frac{\omega^2}{\omega^2\left[1-\left(\frac{\Omega}{\omega}\right)^2\right]} = \frac{1}{1-\lambda^2}$$

$\omega > \Omega$

$$\eta_d = \frac{1}{\lambda^2 - 1}$$

$\omega < \Omega$

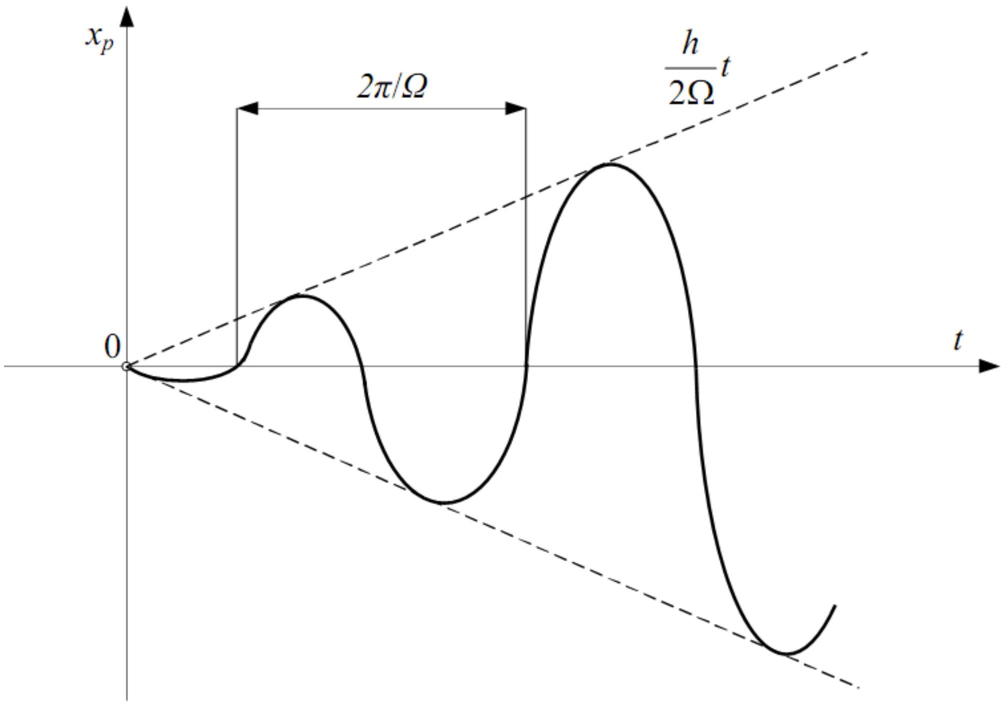
$$(\Omega = \omega)$$

$$x_p = A\,t\cos(\Omega\,t) + Bt\sin(\omega\,t)$$

$$A = -\frac{h}{2\Omega} \qquad B = 0$$

$$x_p = -\frac{h}{2\Omega}t\cos(\Omega t).$$

Equation of motion for resonance



Beating

Beating as phenomena would appear if frequency of the system and exciting force are nearly equal.

During the beating phase of those two movements are changing from 0 to 2π.

$$x = C_1 \cos(\omega t) + C_2 \sin(\omega t) + \frac{h}{\omega^2 - \Omega^2}\sin(\Omega t)$$

$$\frac{\Omega}{\omega} \approx 1$$

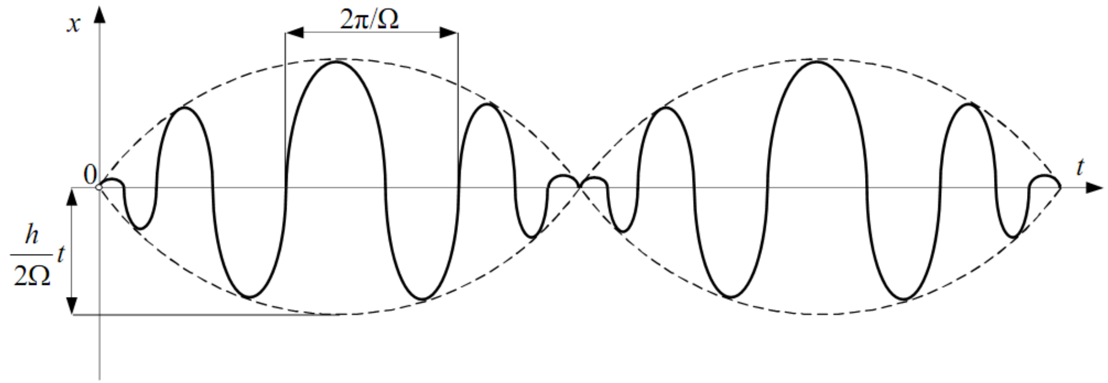
$$0 = C_1, \quad 0 = C_2\omega + \frac{h}{\omega^2 - \Omega^2}$$

$$\omega - \Omega = 2\Delta$$

$$\frac{\Omega}{\omega} = \frac{\omega - 2\Delta}{\omega} \approx 1$$

$$x = \frac{h}{4\Omega\Delta}\cos(\Omega t)\sin(\Delta t)$$

Equation of motion for beating



Damped forced motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\Omega t + \beta)$$

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h \cdot \sin(\Omega t + \beta)$$

$$x_h = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$$

$$x_p = C\sin(\Omega t + \beta - \varphi_0)$$

$$C = \frac{h}{\sqrt{(\omega^2 - \Omega^2)^2 + 4\delta^2\Omega^2}}$$

$$tg\varphi_0 = \frac{2\delta\Omega}{\omega^2 - \Omega^2}$$

$$x = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt) + C\sin(\Omega t + \beta - \varphi_0)$$

Equation of motion for damped forced motion

$\lambda = \frac{\Omega}{\omega}$ · Dimensionless disturbance coefficient

$\psi = \frac{\delta}{\omega}$ · Dimensionless damping coefficient

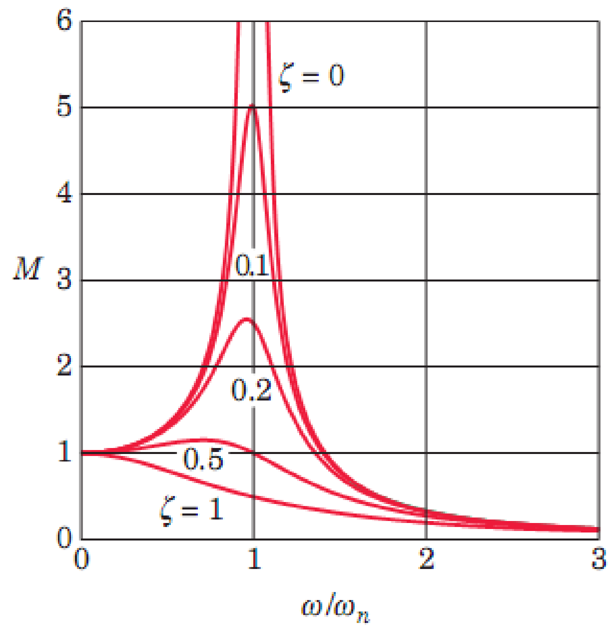
$$\eta_d = \frac{C}{C_{st}} = \frac{1}{\sqrt{(1-\lambda^2)^2 + 4\psi^2\lambda^2}}$$

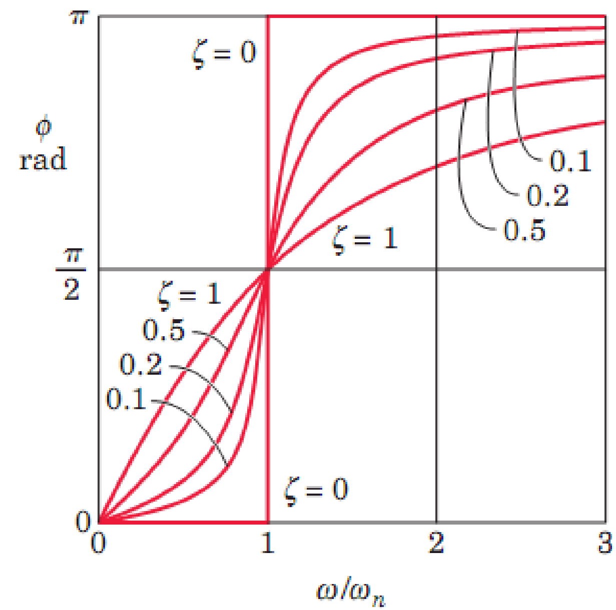
Amplification factor
of damped forced vibrations

$$C = \frac{h}{\omega^2\sqrt{(1-\lambda^2)^2 + 4\psi^2\lambda^2}}$$

$$\frac{h}{\omega^2} = f_{st} = C_{st}$$

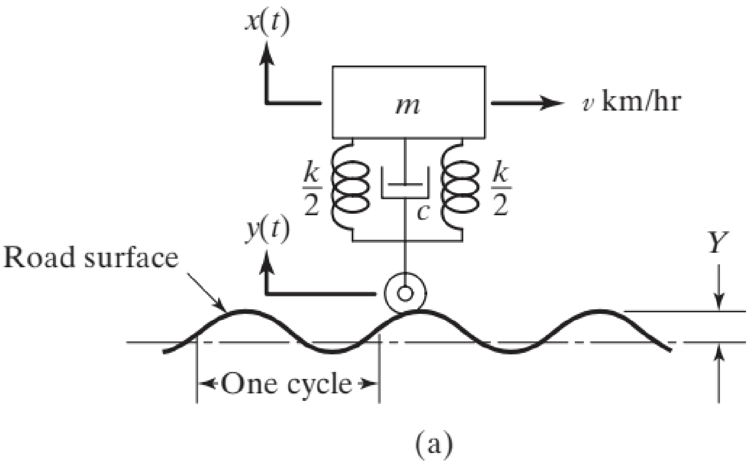
$$\varphi_0 = arctg\frac{2\lambda\varphi}{1-\lambda^2}$$





EXAMPLE:

Figure shows a simple model of a small electrical motor vehicle that can vibrate in the vertical direction while traveling over a rough road. The vehicle has a mass of 1200 kg. The suspension system has a spring constant of 400 kN/m and a damping ratio of ζ= 0.5. If the vehicle speed is 20 km/hr, determine the displacement amplitude of the vehicle. The road surface varies sinusoidally with an amplitude of Y = 0.05 m and a wavelength of 6 m.



(a)

SOLUTION:

$$\omega = 2\pi f = 2\pi\left(\frac{v \times 1000}{3600}\right)\frac{1}{6} = 0.290889v \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{5.81778}{18.2574} = 0.318653$$

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{400 \times 10^3}{1200}\right)^{1/2} = 18.2574 \text{ rad/s}$$

$$\frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{1/2} = \left\{\frac{1 + (2 \times 0.5 \times 0.318653)^2}{(1 - 0.318653)^2 + (2 \times 0.5 \times 0.318653)^2}\right\}^{1/2}$$

$$= 1.100964$$

$$X = 1.100964Y = 1.100964(0.05) = 0.055048 \text{ m}$$