Prinudne oscilacije

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Dinamika i oscilacije

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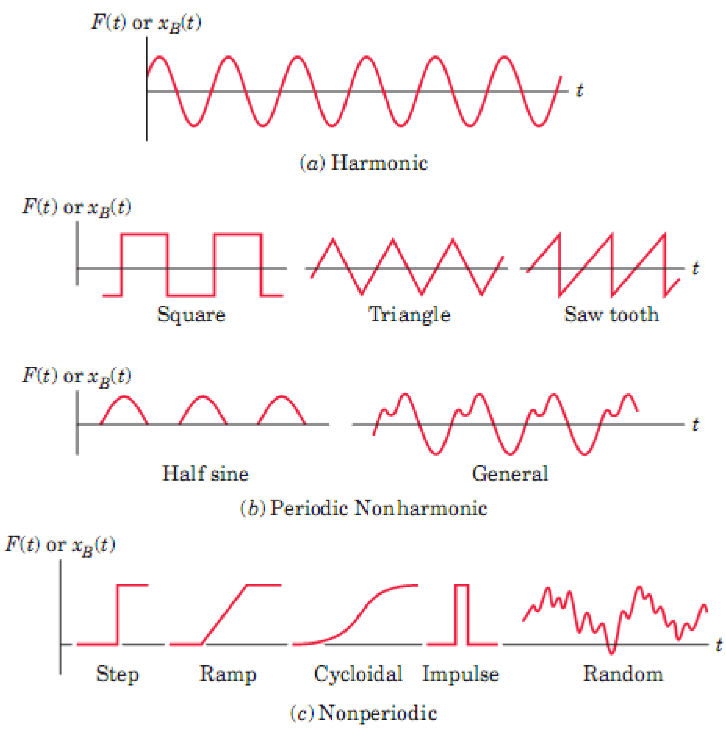
Vrste poremećajne sile



Ukoliko nas sistem djeluje neka spoljašnja sila koja remeti slobodne oscilacije sistema, oscilovanje takvog sistema se naziva prinudno oscilovanje.

Tri su vrste poremećajne sile:

- ***Harmonijska***
- ***Periodična neharmonijska***
- ***Aperiodična***

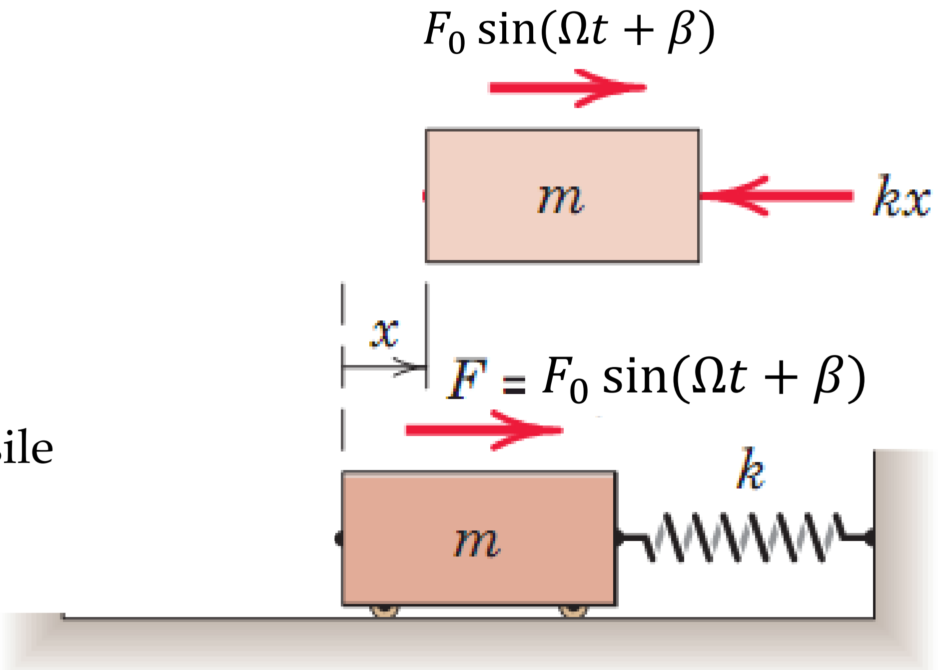
Diferencijalna jednačina prinudnih neprigušenih oscilacija

$$m\ddot{x} = -kx + F_0 \sin(\Omega t + \beta)$$

$$m\ddot{x} + kx = F_0 \sin(\Omega t + \beta)$$

$$\ddot{x} + \omega^2 x = h \sin(\Omega t + \beta)$$

Diferencijalna jednačina prinudnih neprigušenih oscilacija



Ω – Kružna frekvencija poremećajne sile
F_0 – Amplituda poremećajne sile
β – Početna faza poremećajne sile

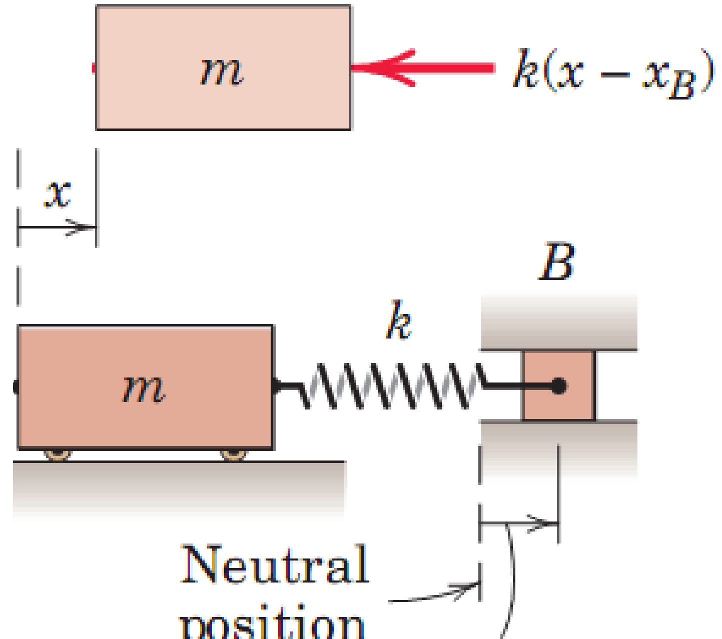
$$h = \frac{F_0}{m}$$

Harmonijsko kretanje oslonca kao prinudna sila

$$m\ddot{x} = -k(x - x_B)$$

$$\ddot{x} + \omega^2 x = \frac{kb}{m}\sin(\Omega t + \beta)$$

Diferencijalna jednačina prinudnih neprigušenih oscilacija uzrokovanih pomijeranjem oslonca



$$x_B = b\sin(\Omega t + \beta)$$

b – amplituda pomijeranja oslonca

Rješenje jednačine kretanja prinudnih neprigušenih oscilacija

$$x = x_h + x_p$$

$$x_h = C_1 \cos \omega t + C_2 \sin \omega t$$

$$\omega \neq \Omega$$

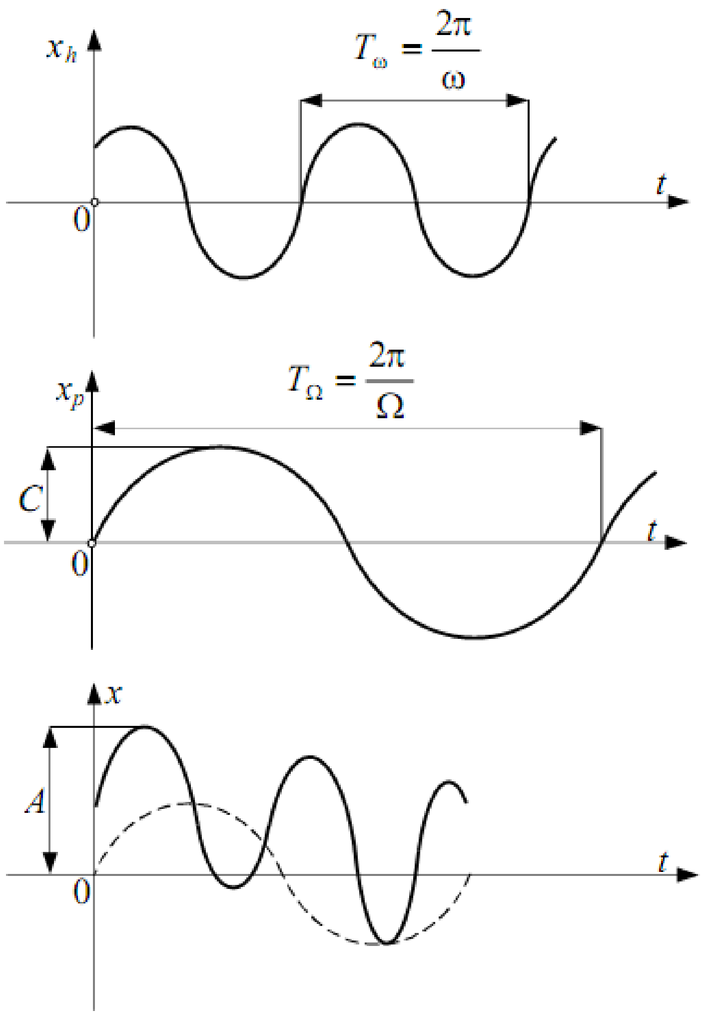
$$x_p = C \sin(\Omega t + \beta) \quad gdje\ je \quad C = \frac{h}{\omega^2 - \Omega^2}$$

$$x_p = \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$

$$x = C_1 \cos \omega t + C_2 \sin \omega t + \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$

ili

$$x = A \sin(\omega t + \alpha) + \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t + \beta)$$



Rezonancija (Ω = ω)

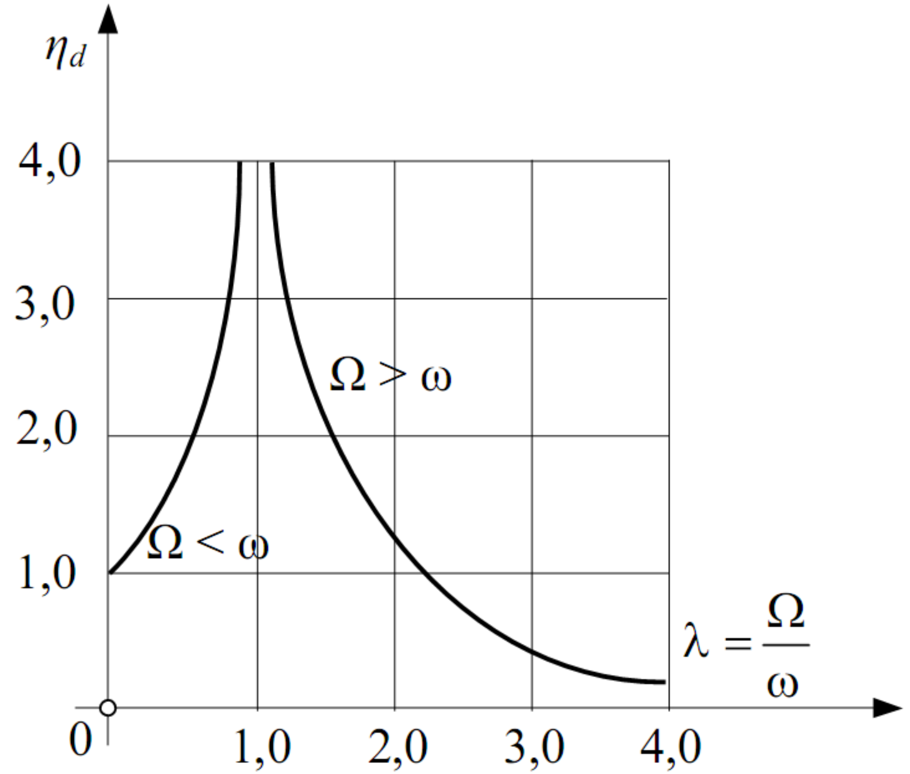
$$x_p = \frac{\frac{h}{\omega^2}}{1-\left(\frac{\Omega}{\omega}\right)^2}\sin(\Omega t + \beta)$$

- statičko izduženje opruge $\frac{h}{\omega^2} = \frac{m \cdot h}{c} = f_{st}$
- koeficijent poremećaja $\lambda = \frac{\Omega}{\omega}$.

dinamički faktor pojačavanja

$$\eta_d = \frac{C_d}{C_{st}} = \frac{\frac{h}{\omega^2 - \Omega^2}}{f_{st}} = \frac{\omega^2}{\omega^2 - \Omega^2} = \frac{\omega^2}{\omega^2\left[1-\left(\frac{\Omega}{\omega}\right)^2\right]} = \frac{1}{1-\lambda^2}$$

$$\eta_d = \frac{1}{\lambda^2 - 1} \qquad \omega < \Omega$$



gdje je ω > Ω

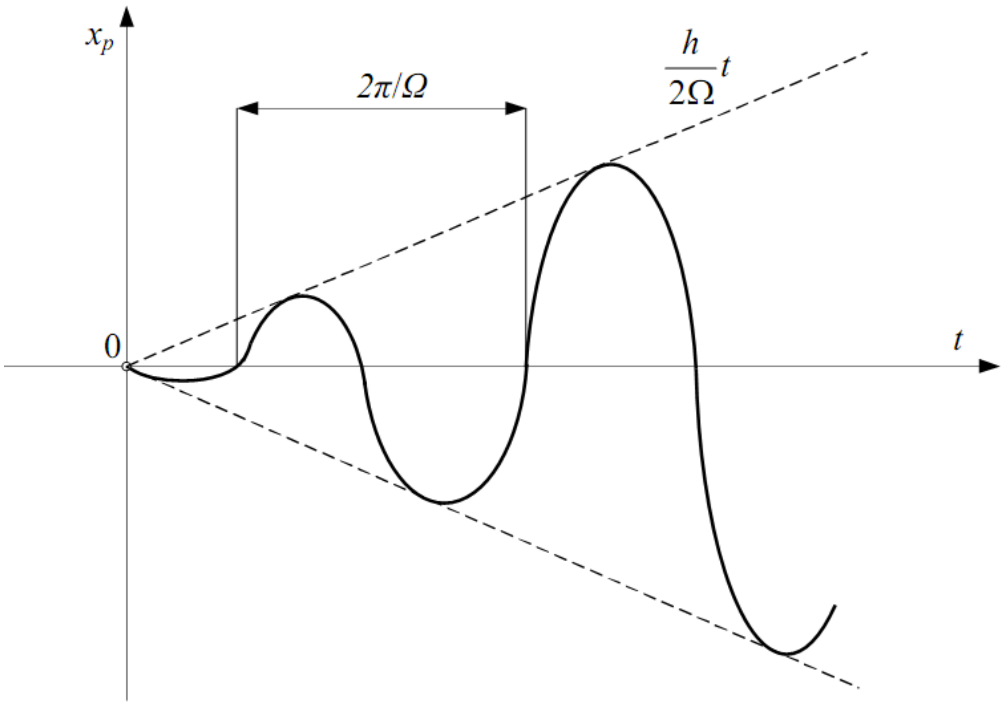
$$(\Omega = \omega)$$

$$x_p = A\,t\cos(\Omega\, t) + Bt\sin(\omega\, t)$$

$$A = -\frac{h}{2\Omega} \qquad B = 0$$

$$x_p = -\frac{h}{2\Omega} t\cos(\Omega t).$$

Zakon prinudnog oscilovanja sistema u slučaju rezonancije



Podrhtavanje

Podrhtavanje se javlja u slučaju kada su kružna frekvencija prinudne sile i kružna frekvencija sistema približne.
Prilikom podrhtavanja faza između ova dva kretanja se mijenja od 0 do 2π.

$$x = C_1 \cos(\omega t) + C_2 \sin(\omega t) + \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t)$$

$$\frac{\Omega}{\omega} \approx 1$$

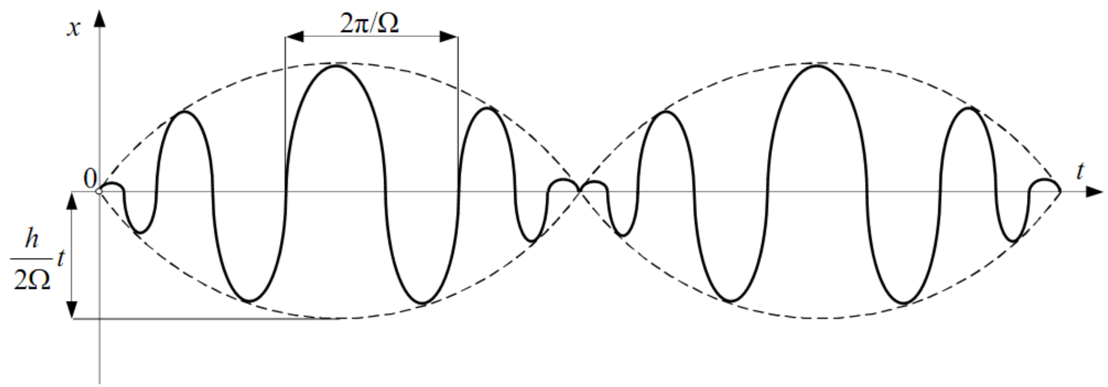
$$0 = C_1, \quad 0 = C_2 \omega + \frac{h}{\omega^2 - \Omega^2}$$

$$\omega - \Omega = 2\Delta$$

$$\frac{\Omega}{\omega} = \frac{\omega - 2\Delta}{\omega} \approx 1$$

$$x = \frac{h}{4\Omega\Delta} \cos(\Omega t) \sin(\Delta t)$$

Zakon kretanja prinudnog neprigušenog sistema u slučaju podrhtavanja



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Prinudne prigušene oscilacije

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\Omega t + \beta)$$

$$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h \cdot \sin(\Omega t + \beta)$$

$$x_h = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$$

$$x_p = C \sin(\Omega t + \beta - \varphi_0)$$

$$C = \frac{h}{\sqrt{(\omega^2 - \Omega^2)^2 + 4\delta^2\Omega^2}}$$

$$tg\varphi_0 = \frac{2\delta\Omega}{\omega^2 - \Omega^2}$$

$$x = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt) + C \sin(\Omega t + \beta - \varphi_0)$$

Zakon oscilovanja prinudnih prigušenih oscilacija

$\lambda = \frac{\Omega}{\omega}$ – bezdimenzionalni koeficijent poremećaja

$\psi = \frac{\delta}{\omega}$ – bezdimenzionalni koeficijent otpora,

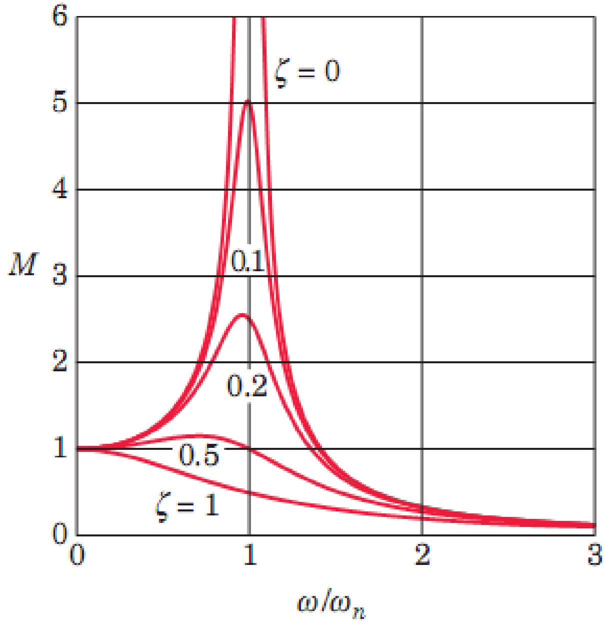
$$C = \frac{h}{\omega^2 \sqrt{(1-\lambda^2)^2 + 4\psi^2\lambda^2}}$$

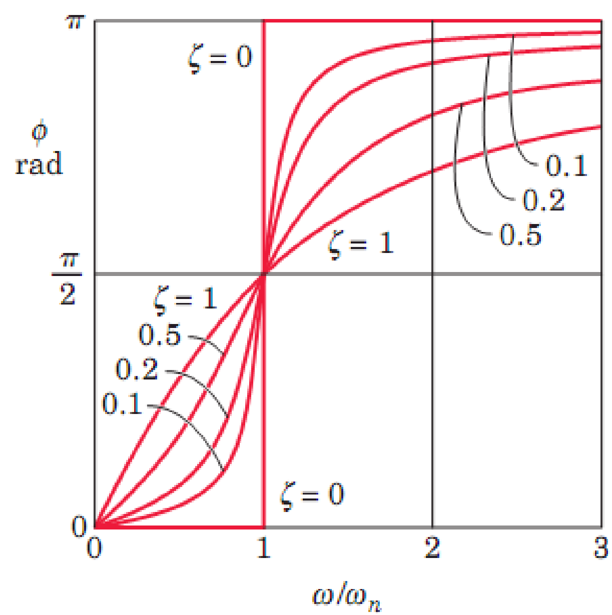
$$\frac{h}{\omega^2} = f_{st} = C_{st}$$

$$\varphi_0 = arctg \frac{2\lambda\varphi}{1-\lambda^2}$$

$$\eta_d = \frac{C}{C_{st}} = \frac{1}{\sqrt{(1-\lambda^2)^2 + 4\psi^2\lambda^2}}$$

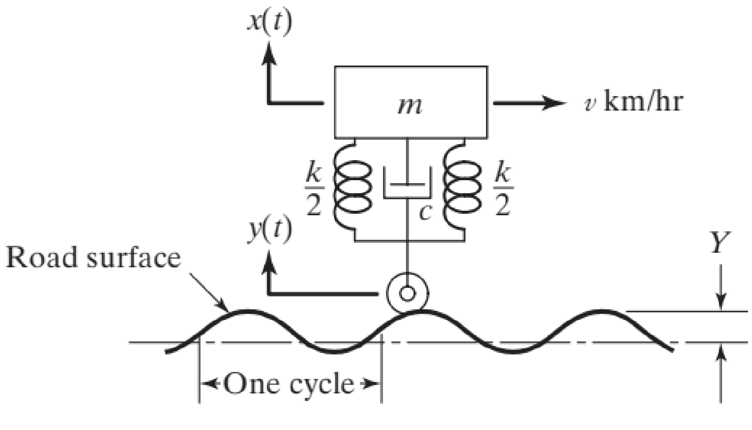
Faktor pojačanja prinudnih prigušenih oscilacija





PRIMJER:

Na slici je prikazan jednostavni model malog električnog voz koji može da osciluje u vertikalnom pravcu dok se kreće po neravnom putu. Masa vozila je 1200 kg. Amortizer se sastoji opruge krutosti 400 kN/m i prigušivača sa faktorom prigušen ζ= 0.5. Ako se vozilo kreće brzinom 20 km/h, Odrediti vertikalnu amplitude pomijeranja vozila. Put se može opisati kao neravnina u obliku sinusoide sa amplitudom Y = 0.05 m I talasnom dužnom (periodom) od 6 m.



(a)

RJEŠENJE:

$$\omega = 2\pi f = 2\pi\left(\frac{v \times 1000}{3600}\right)\frac{1}{6} = 0.290889v \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{5.81778}{18.2574} = 0.318653$$

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{400 \times 10^3}{1200}\right)^{1/2} = 18.2574 \text{ rad/s}$$

$$\frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right\}^{1/2} = \left\{\frac{1 + (2 \times 0.5 \times 0.318653)^2}{(1 - 0.318653)^2 + (2 \times 0.5 \times 0.318653)^2}\right\}^{1/2}$$

$$= 1.100964$$

$$X = 1.100964Y = 1.100964(0.05) = 0.055048 \text{ m}$$