Fluid flow and Bernoulli's equation

Mermina Gadara
Univerzitet Džemal Bijedić

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The amount of fluid flowing in a system per unit time can be expressed in terms of the following three different expressions:

Q-volumetric flow rate
W-Weight flow rate
M-Mass flow rate

- Volumetric flow rate:

$$Q = Av$$

$$Q = Av = \text{m}^2 \times \text{m/s} = \text{m}^3/\text{s}$$

- Weight flow rate:

$$W = \gamma Q$$

$$W = \gamma Q = \text{N/m}^3 \times \text{m}^3/\text{s} = \text{N/s}$$

- Mass flow rate:

$$M = \rho Q$$

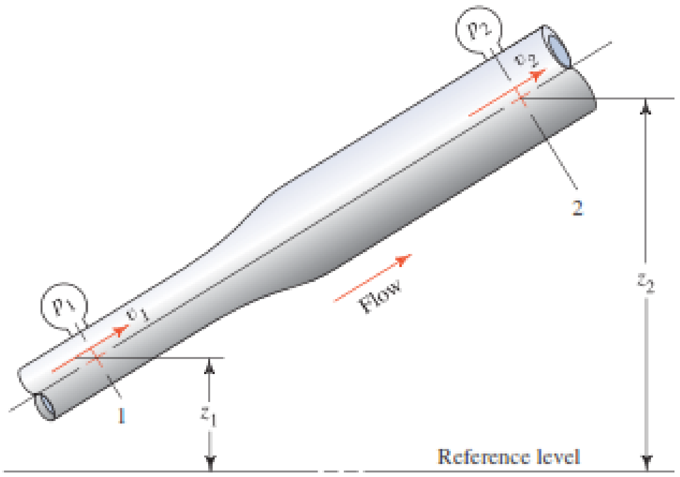
$$M = \rho Q = \text{kg/m}^3 \times \text{m}^3/\text{s} = \text{kg/s}$$

TABLE 6.1 Flow rates—Definitions and units

Symbol	Name	Definition	SI Units	U.S. Customary System Units
Q	Volume flow rate	$Q = Av$	m^3/s	ft^3/s
W	Weight flow rate	$W = \gamma Q$	N/s	lb/s
		$W = \gamma A v$		
M	Mass flow rate	$M = \rho Q$	kg/s	slugs/s
		$M = \rho A v$		

Jednačina kontinuiteta

- The method of calculating fluid flow in a closed pipe system depends on the principle of continuity.



- The amount of fluid that passes through any cross-section at any point in time is constant.

- It can be expressed over:

✓ Mass flow rate:

$$M_1 = M_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

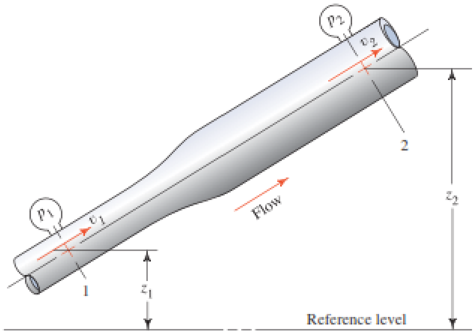
✓ Volumetric flow rate:

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

In Fig. 6.2 the inside diameters of the pipe at sections 1 and 2 are 50 mm and 100 mm, respectively. Water at 70°C is flowing with an average velocity of 8.0 m/s at section 1. Calculate the following:

a. Velocity at section 2
b. Volume flow rate
c. Weight flow rate
d. Mass flow rate



a. Velocity at section 2.

$$A_1 v_1 = A_2 v_2$$

$$v_2 = v_1\left(\frac{A_1}{A_2}\right)$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(50\text{ mm})^2}{4} = 1963\text{ mm}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(100\text{ mm})^2}{4} = 7854\text{ mm}^2$$

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = \frac{8.0\text{ m}}{\text{s}} \times \frac{1963\text{ mm}^2}{7854\text{ mm}^2} = 2.0\text{ m/s}$$

$$Q = A_1 v_1 = 1963\text{ mm}^2 \times \frac{8.0\text{ m}}{\text{s}} \times \frac{1\text{ m}^2}{(10^3\text{ mm})^2} = 0.0157\text{ m}^3\text{/s}$$

$$W = \gamma Q = \frac{9.59\text{ kN}}{\text{m}^3} \times \frac{0.0157\text{ m}^3}{\text{s}} = 0.151\text{ kN/s}$$

$$M = \rho Q = \frac{978\text{ kg}}{\text{m}^3} \times \frac{0.0157\text{ m}^3}{\text{s}} = 15.36\text{ kg/s}$$

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Commercially available pipes

- Steel pipes
- General purpose pipelines are often made of steel.
- Standard sizes are indicated by the Nominal Pipe Size (NPS) and the Schedule Number. The nominal size is just a standard designation and is not used for calculations. The range of layout numbers is from 10 to 160, with larger numbers indicating greater wall thickness.
- The most complete steel pipe series available are Schedule 40 and Schedule 80.

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- Nominal Pipe Sizes (NPS) in Metric Units
  Due to the extensive experience in the production of standard pipes according to the NPS standard, the size and arrangement of numbers are most often used when the pipe system is specified in metric units.

✓ The International Standards Organization (ISO) has developed a way to designate DN.

✓ The symbol DN is used to denote the nominal diameter in mm. Appendix F Displays DN next to NPS.

✓ For example, a DN 50mm Schedule 40 steel pipe has the same dimensions as a 2-inch Schedule 40 steel pipe.

DIMENSIONS OF STEEL PIPE

TABLE F.1 Schedule 40

Nominal Pipe Size		Outside Diameter		Wall Thickness		Inside Diameter			Flow Area	
NPS (in)	DN (mm)	(in)	(mm)	(in)	(mm)	(in)	(ft)	(mm)	(ft^2)	(m^2)
⅛	6	0.405	10.3	0.068	1.73	0.269	0.0224	6.8	0.000 394	3.660×10^{-5}
¼	8	0.540	13.7	0.088	2.24	0.364	0.0303	9.2	0.000 723	6.717×10^{-5}
⅜	10	0.675	17.1	0.091	2.31	0.493	0.0411	12.5	0.001 33	1.236×10^{-4}
½	15	0.840	21.3	0.109	2.77	0.622	0.0518	15.8	0.002 11	1.960×10^{-4}
¾	20	1.050	26.7	0.113	2.87	0.824	0.0687	20.9	0.003 70	3.437×10^{-4}
1	25	1.315	33.4	0.133	3.38	1.049	0.0874	26.6	0.006 00	5.574×10^{-4}
1¼	32	1.660	42.2	0.140	3.56	1.380	0.1150	35.1	0.010 39	9.653×10^{-4}
1½	40	1.900	48.3	0.145	3.68	1.610	0.1342	40.9	0.014 14	1.314×10^{-3}
2	50	2.375	60.3	0.154	3.91	2.067	0.1723	52.5	0.023 33	2.168×10^{-3}
2½	65	2.875	73.0	0.203	5.16	2.469	0.2058	62.7	0.033 26	3.090×10^{-3}
3	80	3.500	88.9	0.216	5.49	3.068	0.2557	77.9	0.051 32	4.768×10^{-3}
3½	90	4.000	101.6	0.226	5.74	3.548	0.2957	90.1	0.068 68	6.381×10^{-3}
4	100	4.500	114.3	0.237	6.02	4.026	0.3355	102.3	0.088 40	8.213×10^{-3}
5	125	5.563	141.3	0.258	6.55	5.047	0.4206	128.2	0.139 0	1.291×10^{-2}
6	150	6.625	168.3	0.280	7.11	6.065	0.5054	154.1	0.200 6	1.864×10^{-2}
8	200	8.625	219.1	0.322	8.18	7.981	0.6651	202.7	0.347 2	3.226×10^{-2}
10	250	10.750	273.1	0.365	9.27	10.020	0.8350	254.5	0.547 9	5.090×10^{-2}
12	300	12.750	323.9	0.406	10.31	11.938	0.9948	303.2	0.777 1	7.219×10^{-2}
14	350	14.000	355.6	0.437	11.10	13.126	1.094	333.4	0.939 6	8.729×10^{-2}
16	400	16.000	406.4	0.500	12.70	15.000	1.250	381.0	1.227	0.1140
18	450	18.000	457.2	0.562	14.27	16.876	1.406	428.7	1.553	0.1443
20	500	20.000	508.0	0.593	15.06	18.814	1.568	477.9	1.931	0.1794
24	600	24.000	609.6	0.687	17.45	22.626	1.886	574.7	2.792	0.2594

TABLE F.2 Schedule 80

Nominal Pipe Size		Outside Diameter		Wall Thickness		Inside Diameter			Flow Area	
NPS (in)	DN (mm)	(in)	(mm)	(in)	(mm)	(in)	(ft)	(mm)	(ft^2)	(m^2)
⅛	6	0.405	10.3	0.095	2.41	0.215	0.017 92	5.5	0.000 253	2.350×10^{-5}
¼	8	0.540	13.7	0.119	3.02	0.302	0.025 17	7.7	0.000 497	4.617×10^{-5}
⅜	10	0.675	17.1	0.126	3.20	0.423	0.035 25	10.7	0.000 976	9.067×10^{-5}
½	15	0.840	21.3	0.147	3.73	0.546	0.045 50	13.9	0.001 625	1.510×10^{-4}
¾	20	1.050	26.7	0.154	3.91	0.742	0.061 83	18.8	0.003 00	2.787×10^{-4}
1	25	1.315	33.4	0.179	4.55	0.957	0.079 75	24.3	0.004 99	4.636×10^{-4}
1¼	32	1.660	42.2	0.191	4.85	1.278	0.106 5	32.5	0.008 91	8.278×10^{-4}
1½	40	1.900	48.3	0.200	5.08	1.500	0.125 0	38.1	0.012 27	1.140×10^{-3}
2	50	2.375	60.3	0.218	5.54	1.939	0.161 6	49.3	0.020 51	1.905×10^{-3}
2½	65	2.875	73.0	0.276	7.01	2.323	0.193 6	59.0	0.029 44	2.735×10^{-3}
3	80	3.500	88.9	0.300	7.62	2.900	0.241 7	73.7	0.045 90	4.264×10^{-3}
3½	90	4.000	101.6	0.318	8.08	3.364	0.280 3	85.4	0.061 74	5.736×10^{-3}
4	100	4.500	114.3	0.337	8.56	3.826	0.318 8	97.2	0.079 86	7.419×10^{-3}
5	125	5.563	141.3	0.375	9.53	4.813	0.401 1	122.3	0.126 3	1.173×10^{-2}
6	150	6.625	168.3	0.432	10.97	5.761	0.480 1	146.3	0.181 0	1.682×10^{-2}
8	200	8.625	219.1	0.500	12.70	7.625	0.635 4	193.7	0.317 4	2.949×10^{-2}
10	250	10.750	273.1	0.593	15.06	9.564	0.797 0	242.9	0.498 6	4.632×10^{-2}
12	300	12.750	323.9	0.687	17.45	11.376	0.948 0	289.0	0.705 6	6.555×10^{-2}
14	350	14.000	355.6	0.750	19.05	12.500	1.042	317.5	0.852 1	7.916×10^{-2}
16	400	16.000	406.4	0.842	21.39	14.314	1.193	363.6	1.117	0.1038
18	450	18.000	457.2	0.937	23.80	16.126	1.344	409.6	1.418	0.1317
20	500	20.000	508.0	1.031	26.19	17.938	1.495	455.6	1.755	0.1630
24	600	24.000	609.6	1.218	30.94	21.564	1.797	547.7	2.535	0.2344

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- Standard steel pipes are used in liquid power systems, capacitors, heat exchangers, fuel engine systems, and industrial liquid process systems.
- The standard inch size is determined by the outer diameter and wall thickness in inches.
- Standard sizes from 1/8 in to 2-in are given in the Appendix.

Recommended flow rates and sizes for pipes

- There are many factors that influence the selection of a satisfactory velocity and flow rate in fluid systems. Some of the most important are the type of fluid, the length of the flow system, the type of pipe, the pressure drop that can be tolerated, the devices (e.g. pumps, valves, etc.) that can be connected to the pipe, temperature, pressure and noise.
- According to the continuity equation, it is known that the flow rate increases as the surface area decreases. Therefore, smaller pipes cause higher speeds, and larger pipes will provide lower speeds.
- Later it will be explained that the energy losses and the corresponding pressure drop increase dramatically as the flow rate increases. For this reason, it is advisable to keep speeds low.

- Since larger pipes are more expensive, some restrictions are required.

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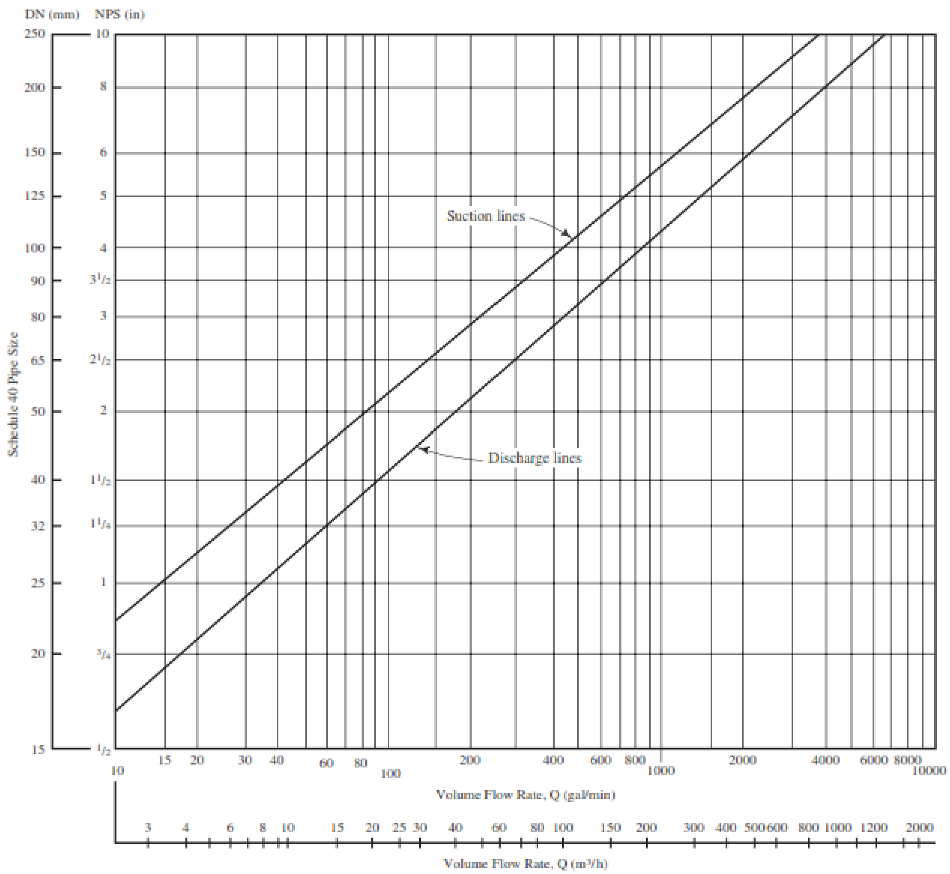

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- The figure provides very rough guidelines for specifying pipe size depending on volumetric flow for typical pumped fluid distribution systems. The data is summarized from the nominal volume flow analysis for many commercially available centrifugal pumps operating close to their best efficiency point and observation of the suction and thrust lines.
- In general, the flow rate is slower in the suction lines that provide flow to the pump to ensure that the suction inlet is properly filled.
- The slower speed also helps limit energy losses in the suction line, keeping the inlet pressure relatively high to ensure that clean liquid enters the pump.
- Lower pressures can cause a harmful condition called cavitation which results in excessive noise, erosion of the pump and rotor surface.

The resulting flow rate from the recommended pipe sizes as shown in the previous figure is generally lower for smaller pipes and higher for larger pipes, as shown in the following data.

Volume Flow Rate		Suction Line			Discharge Line		
		Pipe	Velocity		Pipe	Velocity	
gal/min	m^3/h	Size (in)	ft/s	m/s	Size (in)	ft/s	m/s
10	2.3	1	3.7	1.1	¾	6.0	1.8
100	22.7	2½	6.7	2.0	2	9.6	2.9
500	114	5	8.0	2.4	3½	16.2	4.9
2000	454	8	12.8	3.9	6	22.2	6.8

Recommended flow rates for specialized systems:

- The data in the previous figure refer to the general distribution of fluids in the systems.
- It is advised to look for other information for which the fluid flow is designed.
- For example: the recommended flow rate for a fluid in electric power systems is as follows

	Recommended Range of Velocity	
Type of Service	ft/s	m/s
Suction lines	2–4	0.6–1.2
Return lines	4–13	1.5–4.0
Discharge lines	7–25	2.1–7.6

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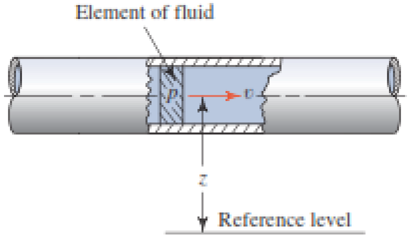
- The suction line delivers fluid from the reservoir to the inlet of the pump.
- The exhaust line carries the high-pressure fluid from the pump outlet to the operation of components such as: actuators or fluidized bed motors.
- The return line transfers fluid from the actuator, pressure relief valve, or engine fluid back to the reservoir.

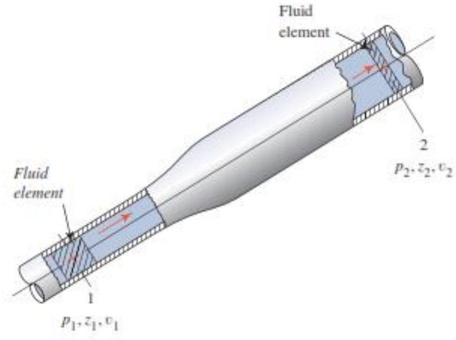
Energy transform

- It is known from physics that energy cannot be destroyed or disappeared, it can only be transformed from one form to another.
- There are three forms of energy that are considered when analyzing the problem of flow in pipes.
- There are three forms of energy possessed by the fluid element:
- Potential energy: $PE = wz$
  - Kinetic energy: $KE = wv^2/2g$
  - Flow energy or pressure energy: $FE = wp/\gamma$
  - Where: $w = \gamma V$



- The total energy is equal to the sum of these three forms of energy:

$$E = FE + PE + KE$$
$$E = wp/\gamma + wz + wv^2/2g$$



- In section 1 total energy is :

$$E_1 = \frac{wp_1}{\gamma} + wz_1 + \frac{wv_1^2}{2g}$$

- In section 2 total energy is :

$$E_2 = \frac{wp_2}{\gamma} + wz_2 + \frac{wv_2^2}{2g}$$

- According to the principle of energy transformation, it follows:

$$E_1 = E_2$$

$$\frac{wp_1}{\gamma} + wz_1 + \frac{wv_1^2}{2g} = \frac{wp_2}{\gamma} + wz_2 + \frac{wv_2^2}{2g}$$

- By arranging the above expression, Bernoulli's equation is obtained:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

Interpretacija Bernulijeve jednačine

- Svaki pojam u Bernoullijevoj jednačini jedan je od oblika energije koju posjeduje fluid po jedinici težine fluida koji teče u sistemu.
- Jedinica za svaki pojam je "energija po jedinici težine".
- U izrazu za Bernulijevu jednačinu razlikju se sljedeće visine:
  - Pritisna visina: p/γ
  - Visina elevacije: z
  - Brzinska visina: $v^2/2g$
- Suma ove tri visine predstavlja ukupnu visinu.

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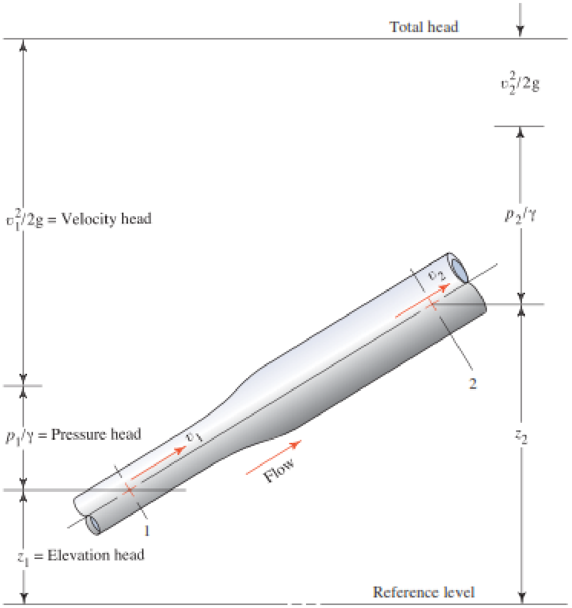
- ✓ Bernoulli's equation explains the changes in pressure, velocity, and between two points in a fluid flow system.
- ✓ It is assumed that there are no losses or additions in energy between them, so the total height remains constant.



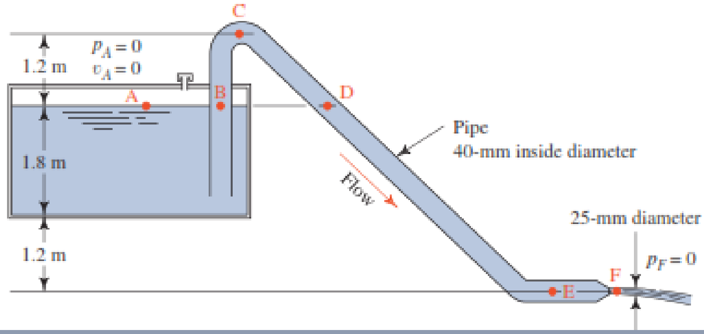
Limitations of the Bernoulli equation

- Although Bernoulli's equation is applicable to solving many problems, there are several limitations that must be well understood in order to be applied correctly:
- It is only valid for incompressible fluids because the specific gravity of the fluid is assumed to be equal to any two cross-sections.
  - Between them there can be no device that can add or remove energy from the system, since according to the equation the energy in the system is constant.
  - There is no heat transfer in or out of the fluid.
- None of the systems satisfies all these limitations in reallity
- The use of this form of the Bernoulli equation is permissible only if a quick rough estimate is required.

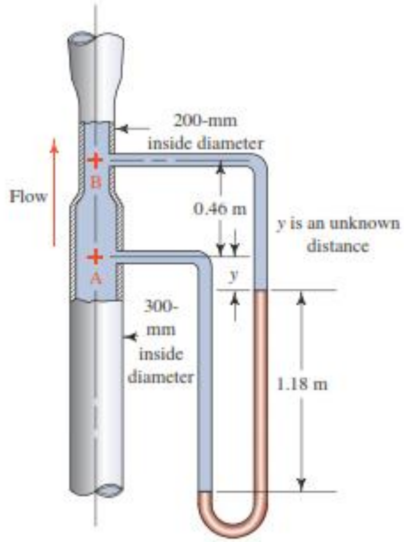
Application of Bernoulli's equation:

➢ **Tanks and nozzles exposed to the atmosphere**

- When a fluid is exposed to atmospheric pressure, the pressure is zero, and can be canceled out of Bernoulli's equation;
- velocity on the surface of the reservoir is assumed to be zero, and can be reversed from Bernoulli's equation;;
- When both reference points are in the same tube (B-E), then the velocity at both points is the same and can be canceled out in Bernoulli's equation;
- When two reference points are at the same level (elevation), then the elevation at the two points is the same, and can be reversed from Bernoulli's equation;



➢Venturimeters and other closed systems with unknown velocities



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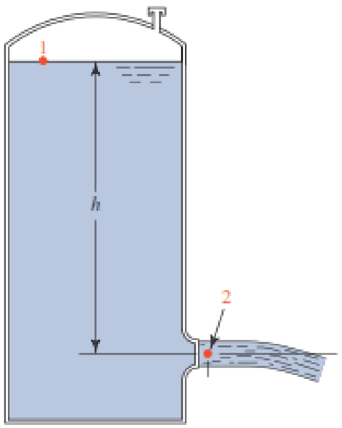
Toricelli's theorem

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$\cancelto{0}{\frac{p_1}{\gamma}} + z_1 + \cancelto{0}{\frac{v_1^2}{2g}} = \cancelto{0}{\frac{p_2}{\gamma}} + z_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g(z_1 - z_2)}$$



Substituting with:

$$h = (z_1 - z_2)$$

$$v_2 = \sqrt{2gh}$$