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Wind Energy As A Renewable Energy Source

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Wind Turbine Power Calculation

- **SCENARIO**

- Wind turbines work on the principle of converting the kinetic energy of the wind into the kinetic energy of the turbine, and then through a generator into electricity, which is further sent through the network to the consumer.

- The total amount of energy that can be converted into electricity depends on the wind speed and the diameter of the turbine. When designing a wind farm, it is important to know the expected power and energy at the output of each of the wind turbines in order to calculate the economic justification.

- **CASE STUDY**

- As we already know, it is crucial to know the power and thus the energy produced by different types of turbines under different conditions. In this example, we will calculate the rotational kinetic energy on a wind turbine at a certain wind speed. This is the minimum wind speed at which the wind turbine produces nominal power.

Wind Turbine Power Calculation

MATHEMATICAL MODEL

- Variables definition

- E = Kinetic Energy (J)
- m = Mass (kg)
- v = Wind Speed (m/s)
- P = Power (W)
- dm/dt = Mass flow rate (kg/s)
- dE/dt = Energy Flow Rate (J/s)

ρ = Density (kg/m3)

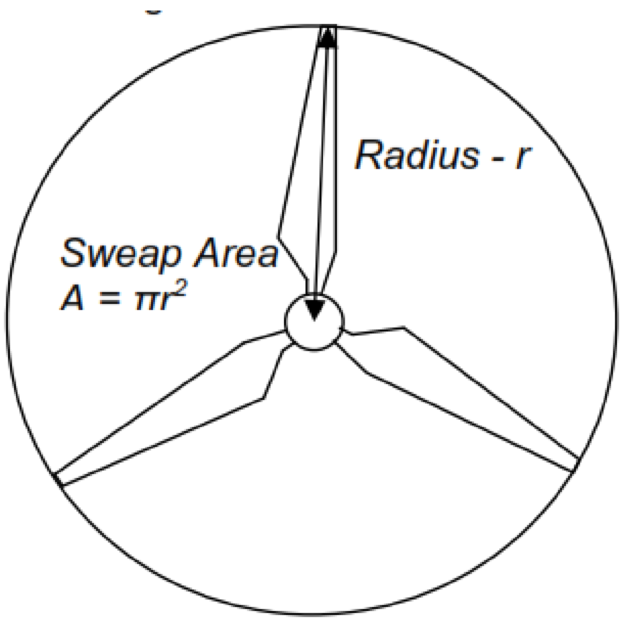
A = Swept Area (m2)

Cp = Power Coefficient

r = Radius (m)

x = distance (m)

t = time (s)



Wind Turbine Power Calculation

MATHEMATICAL MODEL

- At constant acceleration, the kinetic energy of the observed object of mass m and velocity **v** is equal to the work expended **W** in moving the observed body from a state of rest to some distance **s** under the action of force **F**

$$E = W = Fs$$

II Newton law:

$$F = ma$$

It follows

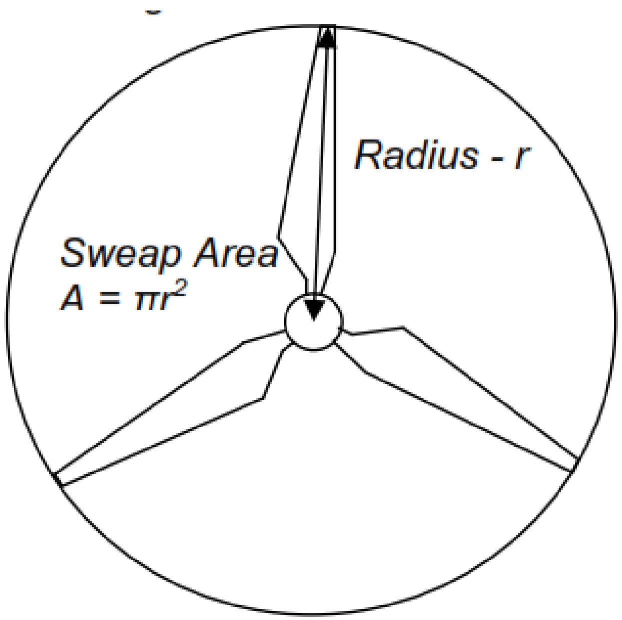
$$E = mas$$

Usinin relation for constant acceleration motion

$$v^2 = u^2 + 2as$$

It becomes

$$a = \frac{(v^2 - u^2)}{2s}$$



Wind Turbine Power Calculation

MATHEMATICAL MODEL

Since the initial velocity of the body is equal to zero u = 0, we get

$$a = \frac{v^2}{2s}$$

By substituting the value of a in equation (1), we get the expression for the kinetic energy of the moving body

$$E = \frac{1}{2}mv^2 \quad \dots (2)$$

Wind power is defined by the rate of change in energy

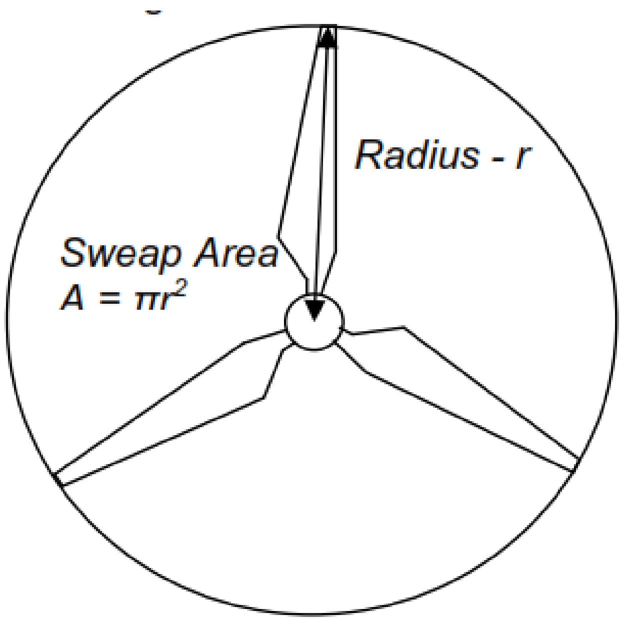
$$P = \frac{dE}{dt} = \frac{1}{2}v^2\frac{dm}{dt} \quad \dots (3)$$

Mass flow is defined by the expression

$$\frac{dm}{dt} = \rho A\frac{dx}{dt} \qquad ; dx/dt=v$$

It follows that

$$\frac{dm}{dt} = \rho Av$$

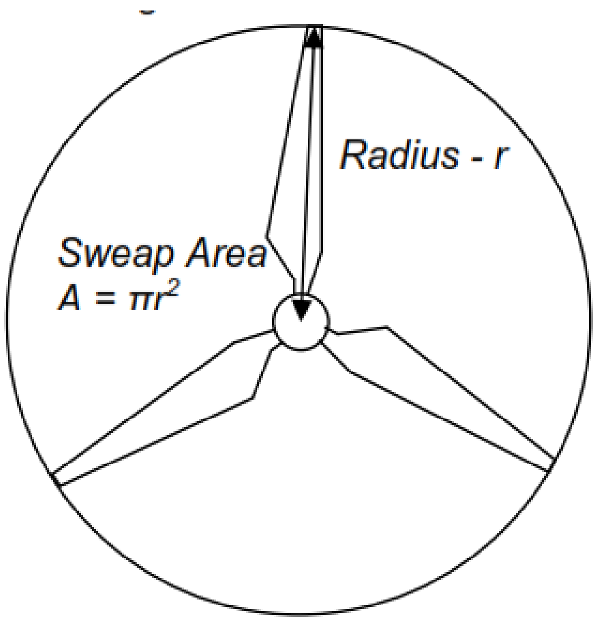


Wind Turbine Power Calculation

MATHEMATICAL MODEL

Based on Equation (3), the wind power is defined as

$$P = \frac{1}{2}\rho A v^3 \quad \dots (4)$$



- In 1919, the German physicist Albert Betz concluded that no wind turbine could convert more than 16/27 (59.3%) of the kinetic energy of the wind into the mechanical energy of the rotor's rotation.
- The Betz Limit or Betz's Law
- The theoretical maximum coefficient of wind power utilization of any type of wind turbine is 0.59
  We call it the power coefficient

$$C_{p_{max}} = 0.59$$

- Wind turbines cannot operate at the maximum limit

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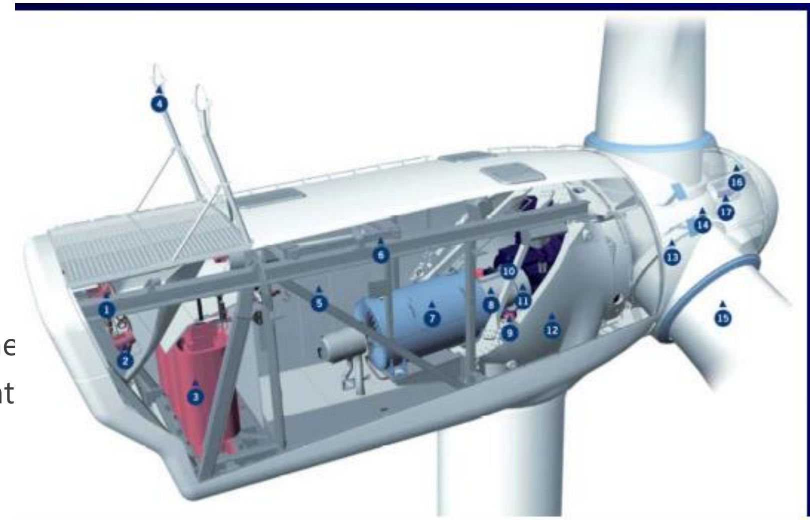
Wind Turbine Power Calculation

The value of the Cp coefficient is unique for each type of turbine, it also depends on the wind speed at which the turbine operates.

When we take into account other requirements such as strength and service life, then the value of the Cp coefficient goes significantly beyond the Betz limit and ranges from 0.35-0.45 even in the best designed wind turbines.

If we take into account the other components of the System, losses on the multiplier, bearings, generator, the actual wind power that is converted int usable energy does not exceed 10-30% of the value of wind power.

From the above, it is evident that the coefficient Cp must be taken into account and added to the expression (4) in order to obtain the expression for the useful force of the wind and it is given by the expression (5)



$$P = \frac{1}{2}\rho A v^3 \quad \dots (4)$$

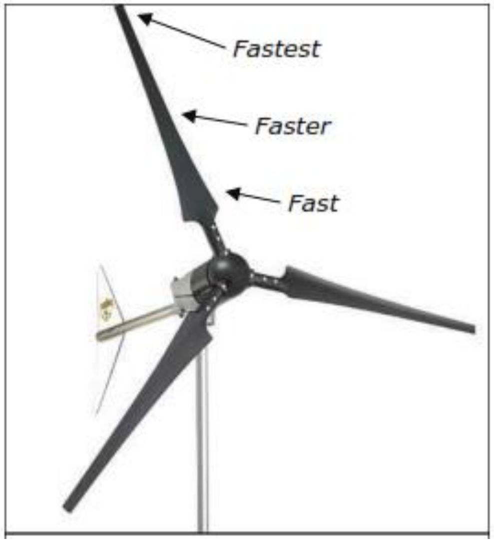
$$P_{avail} = \frac{1}{2}\rho A v^3 C_p \quad \dots (5)$$

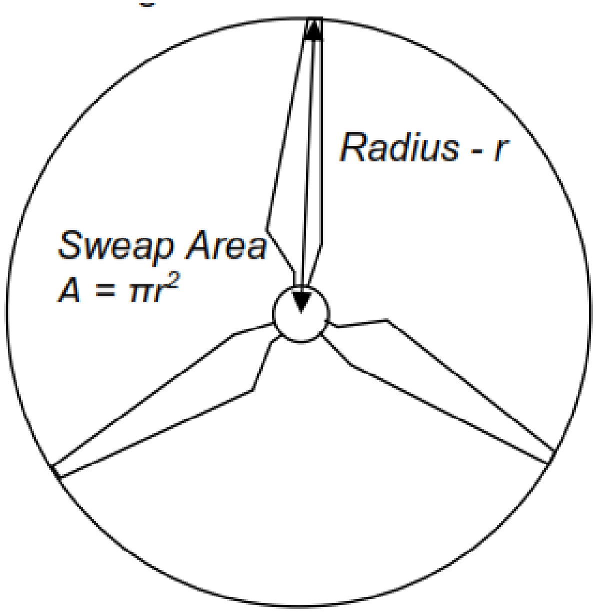
Wind Turbine Power Calculation

The area of the rotor (sweap area) can be calculated from the circle area expression

$$A = \pi r^2 \quad \dots (6)$$

Where radius is equal to blade lenght





EXAMPLE

IT IS GIVEN

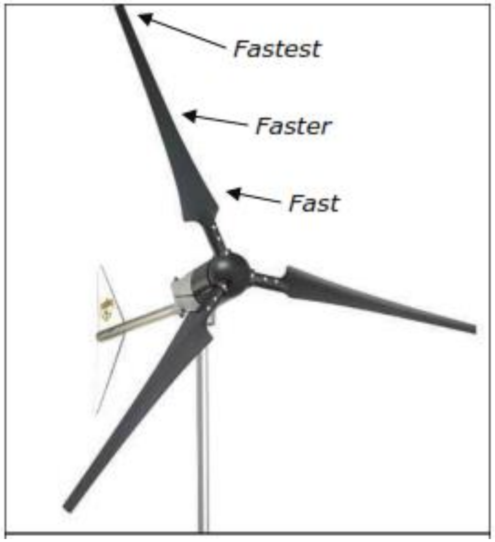
Blade Lenght, l=52 m

Wind velocity, v=12 m/s

Air density, $\rho = 1.23$ kg/m3

Power coefficient Cp=0.4

Calculate the wind energy transferred to the wind turbine rotor!



EXAMPLE

SOLUTION

Blade Lenght, l=52 m

Wind velocity, v=12 m/s

Air density, $\rho = 1.23$ kg/m3

Power coefficient Cp=0.4

$$A = \pi r^2 \ldots (6)$$

Placing the values in equation(6), where l=r ;

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 52^2 \\ &= 8495 m^2 \end{aligned}$$

From the equation defining available power,

$$\begin{aligned} P_{avail} &= \frac{1}{2}\rho A v^3 C_p \\ &= \frac{1}{2} \times 1.23 \times 8495 \times 12^3 \times 0.4 \\ &= 3.6 MW \end{aligned}$$

CONCLUDING CONSIDERATIONS

The resulting value in the previous task is usually defined by the designers of wind turbines

It is important to understand the relationship between the given factors and use the derived equation to calculate the turbine power at wind speeds that differ from the nominal wind speed.

It is important to understand the behavior of the wind turbine at different wind speeds so that we can calculate the losses (€) when the wind turbine is not producing electricity.

It is important to calculate how much energy will be produced in terms of the energy market. Usually, energy is sold before it is produced

Reliable energy production calculations are essential for balancing energy in the market as well as for predicting income for the investor.

EXERCISE-1

1. Repeat the calculation from the previous example for different wind speed values and show a diagram of the power produced for the given wind speeds
2. What is the relationship between wind speed v and power P?
3. Note: in this exercise, we assume that the value of the power coefficient Cp is constant. We know that in reality Cp is a function of wind speed, which will be discussed in more detail in exercise-3

EXERCISE-2

1. Marine turbines are constructed using the same principle as wind turbines. However, they are used in different conditions, where the variables given by equation (5) differ slightly. As marine turbines are powered by water and not air, it is necessary to take water in a specific density instead of air

$$\rho_w = 1000 \text{ kg/m}^3$$

2. The average power coefficient of marine turbines differs from the power coefficient of wind turbines. To date, technology has not advanced to such an extent that the efficiency coefficient of offshore turbines is equal to the coefficient of power of wind turbines. The theoretical maximum is defined by Betz's law of 0.59

3. In the example, the value of the power coefficient Cpm=0.35 will be used

Based on the information provided, modify Equation (5) to include the variables related to marine turbines and calculate the length of the blade required to produce the same power produced by the wind turbine in the previous example. Suppose the pitch is v=2.5 m/s, which is the most common value of the velocity of a 'tidal wave'.

EXERCISE-3

The power coefficient Cp is not constant, as previously and emphasized. Its value changes with the type speed ratio (TSR). TSR is defined as:

$$\lambda = \text{blade tip speed/wind speed}$$

Brzina vrha lopatice može se izračunati kao:

$$\text{blade tip speed} = [\text{angular velocity(rpm)} \times \pi \times D] / 60;$$

where je D-rotor diameter

The angular velocity of the turbine is given to be 15 rpm based on the above two equations. Calculate and then fill in the table given on the next slide. Then read the values of Cp using the given diagram. Use the value of Cp to calculate the power using equation (5).

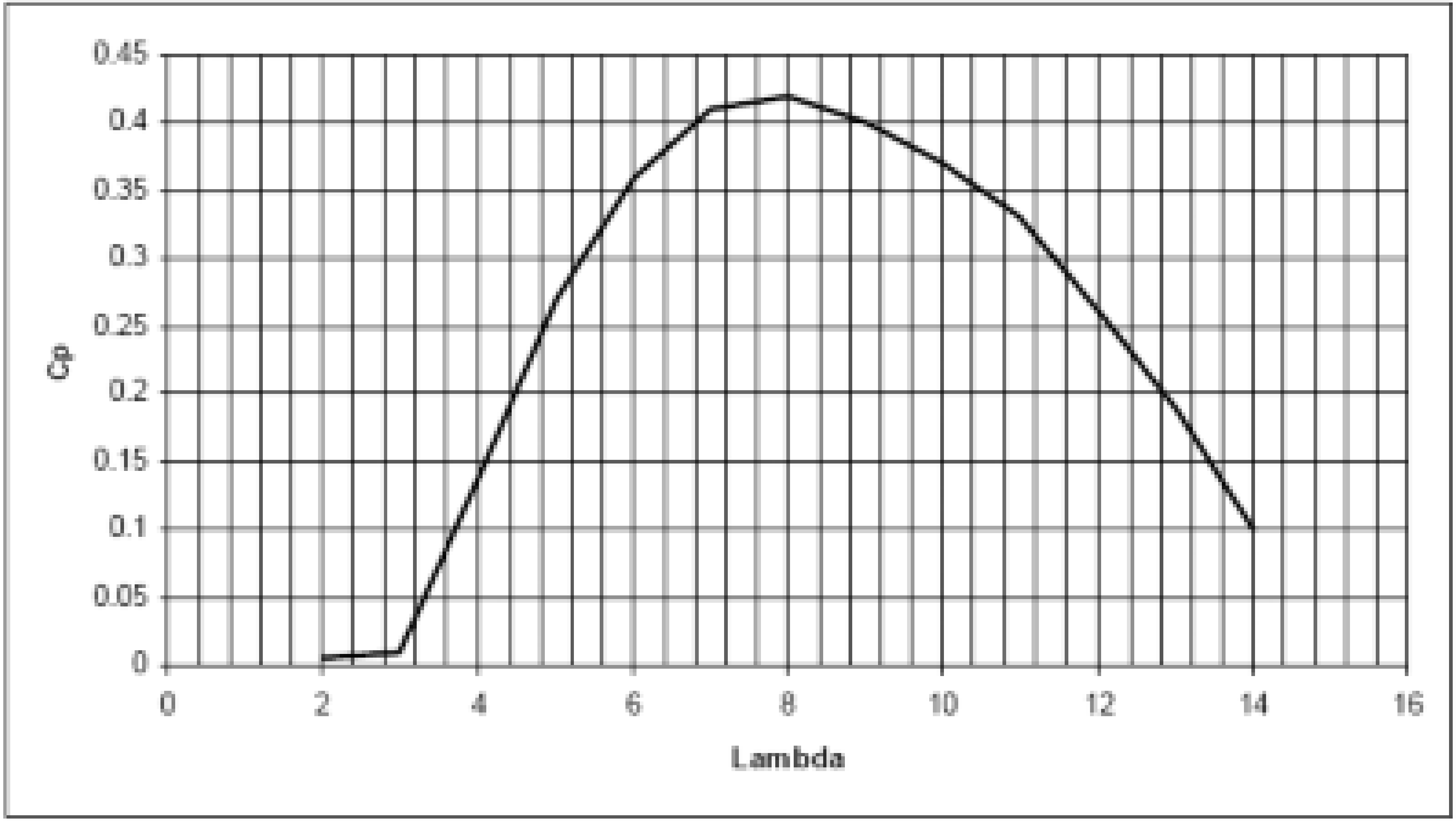
Finally, calculate the energy based on the equation:

$$\text{Enery=Power x Time}$$

Note: The cells crossed out in the table refer to the wind speed values at which the wind turbine does not produce electricity because the values are either low or high.

EXERCISE-3

Wind Speed (m/s)	Time (hours)	λ value	C_p value	Power (kW)	Energy (kWh)
1	531				
3	1407				
5	1831				
7	1769				
9	1386				
11	913				
13	524				
15	249				
17	105				
19	39				
21	12				
23	3				
25	1				
27	0				
Total					

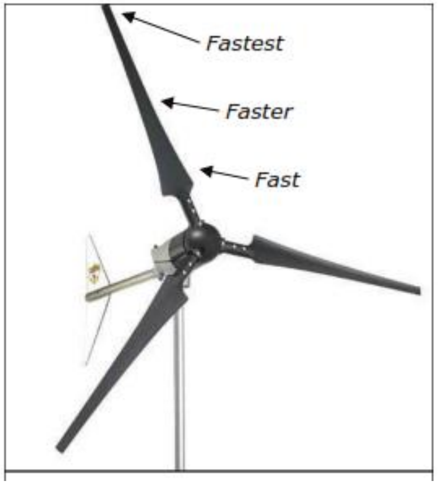

0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Cp
0
2
4
6
8
10
12
14
16
Lambda

BLADE TIP SPEED

If the rotor of the wind turbine spins too slowly, most of the wind will pass straight through the gap between the blades, therefore giving it no power!

But if the rotor spins too fast, the blades will blur and act like a solid wall to the wind.

Also, rotor blades create turbulence as they spin through the air. If the next blade arrives too quickly, it will hit that turbulent air. So, sometimes it is actually better to slow down your blades!



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