PELMOB

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Stability of Linear Automatic Control Systems

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CONCEPT OF SYSTEM STABILITY IN AUTOMATIC CONTROL

- **A stable system** is understood to be a system in which, after an initial disturbance, the process asymptotically settles down.

- A system is considered **stable** if the following holds:

$$\lim_{x \to \infty} x_{pr}(t) = 0$$

- The system is **unstable** or **limitedly stable** if the following holds:

$$\lim_{x \to \infty} x_{pr}(t) \neq 0$$

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IMPORTANCE OF SYSTEM STABILITY

- **System stability** in automation is crucial for the reliable and efficient operation of any technological system.
- Stability directly impacts **the performance** of the automated system.
- In systems such as autonomous vehicles, robotic arms in industry, or medical devices, **stability is critical for safety.**
- Stability ensures that systems operate with minimal energy loss, reducing unsafe variations and unnecessary corrections.

STABILITY IN ENGINEERING SYSTEMS

- In automated systems, such as robotic arms, industrial processes, autonomous vehicles, etc., stability is required for the system to safely and efficiently perform tasks.

- In autonomous vehicles, stability is essential for safe vehicle control in changing driving conditions, such as uphill, steep slopes, slippery roads, etc.

- The use of stability theory in automatic control systems allows engineers to design systems capable of addressing challenges in real-world conditions.

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CONTROL SYSTEMS IN ELECTRIC VEHICLES

Acceleration Control

- In electric vehicles, acceleration is controlled using the **electric motor and the voltage control system.**

- **The acceleration control system** manages the motor's output power by adjusting the amount of current supplied to the motor.

- **Acceleration control algorithms** ensure that the vehicle accelerates smoothly, without sudden jumps in speed, which is important for driving comfort and safety.

Braking Control

- **The electric brake** in an EV, as well as **regenerative braking**, uses the electric motor to direct energy back into the battery during deceleration.
- **Regenerative braking** allows the conversion of kinetic energy, which would otherwise be lost during braking, back into the battery.
- **Braking control** must be carefully coordinated to ensure the vehicle remains under control and to prevent the loss of stability.

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ROLE OF THE CONTROL SYSTEM IN MAINTAINING VEHICLE STABILITY

- Stability Control during Speed Changes
  - Managing excessive acceleration
- Stability Control in Slippery Conditions
  - Traction control, ABS systems
- Stability Control during Sudden Direction Changes
  - Stability control systems (ESP)
- Weight Distribution and Management
  - EVs often have a more favorable weight distribution, which enhances stability

ROUTH STABILITY CRITERION

- The characteristic equation of the system is observed:

$$f(s) = a^n s^n + a^{n-1} s^{n-1} + \cdots + a^1 s^1 + a^0 = 0$$

- For higher-order systems, one of the stability assessment criteria must be applied.
- Two algebraic criteria were independently established by Routh and Hurwitz at the beginning of the 19th century.
- The criterion is set with the aim of determining the nature of the solutions to the characteristic equation (the sign of the real part of all the solutions to the equation), without solving the equation itself.

Routh criterion

- The characteristic equation is observed.
- Based on the coefficients of the characteristic polynomial f(s), the Routh array of coefficients is formed according to the following table:

s^n	a_n	a_{n-2}	a_{n-4}	...
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	...
s^{n-2}	b_1	b_2	b_3	...
s^{n-3}	c_1	c_2	c_3	...
$\vdots$				
s^0	h_1			

Routh criterion

- The first two rows of the Routh array consist of the coefficients of the characteristic polynomial.
- The remaining elements, starting from the third row and onwards, are calculated in the following way:

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$b_3 = \frac{a_{n-1}a_{n-6} - a_n a_{n-7}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1}$$

$$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$$

$$\vdots$$

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Routh criterion

- Once the array is formed, the first column – the Routh column – is observed.

- **The necessary and sufficient condition for the system to be stable:**
  - All elements of the Routh column, formed based on the coefficients of the characteristic polynomial, must have the same sign (usually positive).

- The system will be marginally stable if zeros appear in the Routh column alongside coefficients of the same sign.

- The system will be unstable if there are both positive and negative elements in the Routh column.

Hurwitz criterion

- Based on the coefficients of the characteristic polynomial *f(s),* the Hurwitz determinant of the coefficients is formed according to the following table:

$$\Delta_h = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & 0 & 0 \\ a_n & a_{n-2} & a_{n-4} & \cdots & 0 & 0 \\ 0 & a_{n-1} & a_{n-3} & \cdots & 0 & 0 \\ 0 & a_n & a_{n-2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & a1 & 0 \\ 0 & 0 & 0 & & a2 & a0 \end{vmatrix}$$

- **The system will be stable** if all the diagonal minors of the Hurwitz determinant, formed based on the coefficients of the characteristic polynomial, are greater than zero.

Hurwitz criterion

- Therefore, the necessary and sufficient conditions for the stability of a system, according to Hurwitz, are:

$$\Delta_1 = a_{n-1} > 0$$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = a_{n-1}a_{n-2} - a_n a_{n-3} > 0$$

$$\Delta_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} > 0$$

$$\vdots$$

$$\Delta_n = \Delta_h.$$

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Hurwitz criterion

- The system will be **unstable** if some diagonal minors are positive and some are negative.

- The system will be **marginally stable** if the last diagonal minor (Δh) is equal to zero, and all previous ones are positive.

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Example of Applying the Routh and Hurwitz Stability Criteria in Electric Vehicle Stability

- Motor Control (Torque Control)
- Regenerative Braking Systems
- Power Distribution Control Between Motor and Battery
- Vehicle Stability Control Systems (ESC - Electronic Stability Control)
- Autonomous Driving Systems in Electric Vehicles
- Suspension Control in Electric Vehicles (Active Suspension)
- …

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